Non-diversifiable Risk and Endogenous Innovation

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December 31, 2018

Job Market Paper
Latest version: https://petukhov.mit.edu

Abstract

I propose a dynamic general equilibrium model to study jointly the process of innovation and asset pricing properties of investment in innovation. Historically, times of high innovation activity have been associated with high levels of idiosyncratic risk both at the firm and individual investor level. The model mirrors this empirical fact and emphasizes both productivity enhancing and disruptive effects of innovation, that endogenously determine the level of idiosyncratic risk. When markets are incomplete, the disruptive nature of innovation gives rise to hedging motives against potential disruption. In aggregate, investors’ hedging activity actually increases the pace of disruptive innovation and generates the observed empirically volatile fluctuations of the volume of and risk premium on investments in innovation. The model reconciles empirical properties and puzzles of aggregate flows and performance in venture capital. Empirical tests, utilizing the cross-section of stocks, confirm asset pricing implications of the model.
1 Introduction

The process of technological innovation is inherently risky. On the one hand, chances of success in innovation are low. On the other hand, technological innovation poses a risk of disruption to operating businesses through the Schumpeterian force of creative destruction. In this paper, I study the role financial risks play in the process of innovation. I propose a general equilibrium model in which agents’ exposures to the risk of disruption and hedging motives shape jointly the nature of technological progress and asset prices.

The starting point to this study is a set of stylized facts associated with innovation and risk at the aggregate economy level. First, innovation activity follows moderately persistent medium-term cycles. Second, aggregate returns on investment in innovation, as measured by performance of the venture capital sector, are highly risky. Third and most important, I show that innovation activity at the aggregate economy level tightly comoves with idiosyncratic risk measured both at the firm level and investor portfolio level. At the same time, there is only a weak relationship between the volatility of the market portfolio and the innovation activity.

I propose a model that mirrors these empirical facts. My model highlights the importance of unpredictable reallocative consequences of innovation that give rise to idiosyncratic risk of disruption, in addition to more commonly studied uncertain improvements to productivity resulting from innovation. For investors holding the fully diversified market portfolio, the process of reallocation might not pose a substantial risk. The risk of disruption is, however, acute for investors, who hold concentrated positions in incumbent firms. Among others, this group of investors includes executives, entrepreneurs, firm founders and family members who own significant undiversified equity stakes in their businesses. Jointly, this group of investors owns a large share of wealth in the population and their portfolio choices have significant consequences for the aggregate economy.

I find that agents’ non-diversified exposure to incumbent firms has crucial effects on the equilibrium amount of innovation activity and its nature. In particular, it magnifies the risk of disruption and exacerbates the business-stealing effect of radical innovation. Agents fail to coordinate and invest excessive amount of resources in disruptive technologies. In the dynamic context, this effect leads investment in innovation to sharply respond to changes in the aggregate economic conditions. As a result, consistent with the data, investment in innovation is highly volatile. The risk premium on investments in innovation is not constant but time-varying and can switch signs. When the economy is in a phase of expansion, investment in radical innovation is high and the risk premium on innovation

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is low. The opposite is true in periods of contraction.

I evaluate quantitative performance of the model by calibrating it to reflect the interplay between the market of publicly traded firms and the venture capital sector. The model reproduces both average returns and volatility of returns in VC; however, it slightly understates volatility of fund flows. In addition, endogenous variation in risk premium helps explain the stylized facts linking aggregate VC flows with the sector’s financial performance, that are well documented in the empirical literature. First, the model replicates strong negative relationship between fund inflows and subsequent returns. Second, qualitatively the model generates the positive relationship between realized returns and following inflows to the venture capital sector.

I conduct additional tests of the asset pricing implications of the model using the cross-section of stocks. I form portfolios based on the number of years since IPO. Firms with more recent IPO are on average younger, and are more likely to benefit from technological developments. Consistent with the theory, I show that returns on the “young-minus-old” portfolio are positively loaded on shocks to idiosyncratic risk. In addition, the level of innovation activity significantly negatively forecasts returns on the “young-minus-old” portfolio. One standard deviation increase in R&D-to-GDP ratio is associated with five percentage point decrease in return on that portfolio in the following year.

My setup builds upon the classical quality ladder framework of Aghion and Howitt (1992) and Grossman and Helpman (1991). In the model, technological progress is volatile and uncertain both at the aggregate and individual firm level. Most of the time the economy grows in a steady manner. Yet occasionally, aggregate innovation waves arrive, causing radical changes in production technologies with heterogeneous impact on individual firms. Some firms stay unaffected by the wave, some succeed in disruptive innovation and enjoy an increase in productivity, while the other are left behind in the technological race and are displaced by new entrants. Historical examples of such waves include inventions of steam and combustion engines, breakthroughs in space exploration, the discovery of semiconductors, or more recently — the development of mobile internet and breakthroughs in machine learning, among many others. The size of each wave depends endogenously on the amount of investment in innovation, and the economy-wide impact of innovation waves is a source of systematic risk that is associated with a significant risk premium.

A crucial departure of this framework from traditional models of growth with quality ladders is that the economy features heterogeneous agents. Agents of the first type, which I call households, can freely choose allocations of wealth across investment opportunities.
In contrast, the second type, which I call managers, bears a non-diversifiable background risk. In particular, each manager is matched with an incumbent firm and is obliged to hold a fixed share of her wealth in the stocks of the managed firm. This assumption captures in a reduced form the moral hazard problem. Hence the manager faces a risk of disruption by an entering firm, which can be realized in case of a wave arrival. The manager can (imperfectly) hedge this risk, first, by investing in disruptive innovation on behalf of the managed firm to outcompete entrants in the technological race and, second, by allocating a bigger share of the portfolio into an innovation sector. The innovation sector is an intermediary that allocates investors’ contributions among startups. Payoffs on investments in the innovation sector positively correlate with successful entry of new firms, and thus such investments offer a good hedge for managers.

Aggregate allocations to the innovation sector determine the amount of disruptive innovation in the economy. At the same time aggregate amount of innovation impacts investors’ hedging behavior. The two-way feedback loop between agents’ portfolio choice and the amount of innovation gives rise to the herding in the innovation effect. Higher innovation activity implies higher probability of disruption of any given firm. Hence, in an attempt to hedge, each incumbent manager contributes even more to the innovation sector. Collectively, through portfolio choice investors increase probability of disruption of any given firm. In my setup, the business-stealing effect of disruptive innovation unambiguously hurts managers’ welfare as a group. Herding in the innovation amplifies this negative effect. This externality, induced by the optimal portfolio choice, is reminiscent of DeMarzo et al. (2004, 2007, 2008). The underlying mechanism is, however, different. In DeMarzo et al. (2004, 2007, 2008) agents compete for limited resources in the incomplete market setup, which leads to relative wealth concerns and under-diversification. In the current setup, agents attempt to diversify their own risk, but do not take into account the higher risk imposed on other agents, which leads to the prisoner’s dilemma scenario.

The sign and the magnitude of the risk premium on innovation depend on the relative strength of two channels. First, improvements to productivity of incumbent firms, that come with arrival of innovation waves, push the premium up. Second, the downside risk, that managers face, drives it down. In my calibration, the risk premium is positive most of the time. It can turn negative, however, in periods of very intensive innovation activity with a high potential for disruption, akin to the one observed during the internet boom of the late 90s.

Both the background risk and lumpy arrival of disruptive innovations are critical ingredients for my results. To show this, first, I gradually reduce the size of stakes that
managers have to keep in their firms. Smaller stake sizes reduce the amount of background risk faced by the agents. Disruptive effects of innovation become less important for pricing the risk of innovation. Both risk premium and flows into the innovation sector become less responsive to the state of the economy. Equilibrium amount of investment in innovation goes down and risk premium goes up.

Second, I vary the rate of arrival of innovation waves while keeping intensity of disruptive innovation arrival for any given firm unchanged. Higher rates of arrival make each wave smaller in size. Disruptive innovations become less lumpy and fluctuations of the price of risk become less pronounced. In the limiting case of infinite intensity, waves arrive at any given instant. The amount of disruptive innovations becomes fully predictable and the innovation sector loses its hedging properties. The price of innovation risk converges to zero.

Related Literature This paper fits into the growing literature that studies the interplay of economic growth and technological change with asset prices and returns. Greenwood and Jovanovic (1999), Bond et al. (2000), Laitner and Stolyarov (2003), Comin et al. (2009) relate evolution of the stock market valuations to technological shifts. Pastor and Veronesi (2009) explore the effect of learning and uncertainty on asset prices caused by arrival of new technologies. Garleanu et al. (2012) study how adoption of technologies shapes consumption and risk premia on the stock market. Andrei and Carlin (2018) explore the effects of competition on equilibrium innovation and stock prices. Closer to this paper, a set of studies endogenize the process of economic growth. Kung and Schmid (2015), Bena et al. (2015) investigate how endogenous innovation can give rise to the medium and long-run risk (Bansal and Yaron, 2004). Loualiche et al. (2014), Corhay et al. (2017) model the interplay between risk premia on stocks and firm entry and exit decisions. Different from this strand of literature, the focus of my paper is not only implications of innovation for the stock market but also the asset pricing properties of investments in innovation.

In this respect, the current study contributes to the emerging literature that ties the macroeconomy with the venture capital industry. Opp (2016) and Greenwood et al. (2018), pioneers of this literature, carefully model micro frictions of the VC sector and investigate the impact of venture capital industry on the stock market, growth rate and welfare in the economy. Complementary to these studies, I focus on the endogenous risk properties of the innovation sector and how this risk shapes the process of innovation and returns on investment in innovation. In a contemporaneous and independent paper, Jovanovic and Rousseau (2018) show that fluctuations in the price of capital, as measured
by Tobin’s Q, can help explain behavior of returns on VC and buyout funds. My approach differs as it focuses on the sources and pricing of risk in an equilibrium framework with no arbitrage.

The key risk channel associated with the process of innovation in my model builds upon the notion of displacement introduced by Gărleanu et al. (2012). Kogan et al. (2016), Garleanu et al. (2015), Gărleanu and Panageas (2017) study implications of displacement for the pricing of aggregate stock market, cross-section of stocks and alternative assets. Distinct from these studies, the displacement effect of innovation in my setup is critical only for a subset of population while the majority of agents benefit from the process of innovation. As a result, the risk premium associated with innovation can be both positive and negative depending on the state of the economy. In this respect my mechanism is tightly linked to the literature on the asset pricing effects of idiosyncratic risk (Constantinides and Duffie, 1996; Heaton and Lucas, 2000; Storesletten et al., 2007; Herskovic et al., 2016; Schmidt, 2016; Constantinides and Ghosh, 2017).

A number of papers study implications of innovation on the evolution of inequality. Aghion et al. (2018) find an empirical link between innovation and top income shares. Jones and Kim (2018) in a Schumpeterian framework show that disruptive innovation gives rise to the Pareto distribution of income. Gomez (2018) focuses on idiosyncratic risk as a source of wealth inequality dynamics. The model, I develop in the current paper, has implications for the wealth inequality driven by technological innovation as well. More importantly, I show the reverse mechanism: wealth concentrated in the hands of non-diversified agents can be a driver of innovation on its own.

From a methodological perspective, my study is closely related to the vast literature on endogenous economic growth, in particular models of innovation by incumbents and entrants (Comin and Mulani, 2009; Acemoglu et al., 2013; Acemoglu and Cao, 2015; Garcia-Macia et al., 2015). In addition, I rely on techniques developed in macroeconomic models with agent heterogeneity (He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Di Tella, 2017).

2 Stylized Facts

Numerous studies have documented that innovation activity at the economy level, measured by R&D expenditures, follows cyclical patterns.2 Campbell et al. (2001), Comin

Comin and Philippon (2005) document that firm-level volatility was trending up during 1950–2000. Comin and Philippon (2005) associate it with increasing R&D-based competition over the time period. In this section I confirm that innovation activity follows medium term cycles. In addition, I show new evidence on comovement patterns between innovation and (i) idiosyncratic risk at the firm level, (ii) idiosyncratic risk at the individual investor level, and (iii) aggregate market risk.

**Firm-level Idiosyncratic Risk**  My preferred measure of the innovation activity is R&D expenses normalized by the GDP. To measure the economy level R&D expenditures I use annual data collected by the National Science Foundation on R&D funded by US domestic businesses. The R&D-to-GDP ratio has been trending up since the beginning of the NSF data sample, hence I extract cyclical component by removing a linear trend and work with the obtained series from now on.

Solid line on Figure 1 plots a rescaled version of the measure of innovation activity $IA_t$. The graph shows that innovation activity follows moderately persistent medium-term fluctuations with several distinct peaks. It was at the highest levels during the technological boom in late 1990s and early 2000s. These medium-term fluctuations seem to be not related to more traditional higher frequency business cycles: innovation activity was declining during three recessionary episodes identified by the NBER and was on the rise during five recessionary episodes in my sample.

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3In an independent paper Gomez (2018) finds a similar relationship between individual investor risk and innovation.

4See Griliches (1994) or more recent Bloom et al. (2017) for the expansive discussion and more references on the trends in R&D volume and productivity.
Table 1: **Innovation activity and firm-level idiosyncratic risk.** The table presents results of linear regressions of measures $Y_t$ of firm-level idiosyncratic risk on innovation activity $IA_t$: $Y_t = \text{Const} + \beta IA_t + \gamma_1 \text{Output Gap}_t + \gamma_2 t + \epsilon_t$. Innovation activity is measured as the detrended ratio of economy-wide R&D/GDP ratio normalized to have zero mean and unit variance. Output gap is normalized to unit variance. Rank Reallocation, CS StDev and Quantile Range are calculated in the sample of public firms present in CRSP, annual data: 1953–2016. Rank Reallocation is defined by equation (1), CS StDev is the cross-sectional standard deviation of annual returns, Quantile Range is the difference between 95%-tile and 5%-tile of firm annual returns in a given year. Job Reallocation is the economy-wide measure reported by Business Dynamics Statistics, annual data 1975–2015. HAC standard errors in parentheses. One, two and three stars denote significance at the 10, 5 and 1% level.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>Rank Reallocation</th>
<th>CS StDev</th>
<th>Quantile Range</th>
<th>Job Reallocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$IA_t$</td>
<td>0.010***</td>
<td>0.106***</td>
<td>0.209***</td>
<td>0.014***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.030)</td>
<td>(0.069)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Output Gap</td>
<td>−0.001</td>
<td>−0.045</td>
<td>−0.058</td>
<td>−0.0004</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.042)</td>
<td>(0.066)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Mean($Y_t$)</td>
<td>0.061</td>
<td>0.559</td>
<td>1.431</td>
<td>0.286</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.599</td>
<td>0.368</td>
<td>0.504</td>
<td>0.749</td>
</tr>
</tbody>
</table>

The dash line on Figure 1 denotes a market capitalization based measure of reallocation $R_t$. I construct this measure using the CRSP sample of publicly traded firms headquartered in the US. In year $t$ it is defined by

$$R_t = \frac{\sum_{i\in \text{CRSP}} |Q_{i,t}^{MC} - Q_{i,t-1}^{MC}|}{N}, \quad (1)$$

where $Q_{i,t}^{MC}$ is firm’s $i$ quantile in the market capitalization ranking on the last trading day of year $t$; the sum is calculated over firms for which market capitalization and, hence, the corresponding quantile are observed in both years $t$ and $t − 1$; $N$ is the number of firms in the summation. In a hypothetical scenario, when firms’ capitalization changes in the same proportion across firms from year $t$ to year $t − 1$, this measure is zero. In presence of idiosyncratic risk, firms’ capitalization ranking changes from one year to another and, hence, $R_t$ is positive. As we see on Figure 1, $R_t$ follows a pattern very similar to the innovation activity and strongly positively comoves with it. Correlation between the two series is 0.62.

5Due to year-to-year changes in the CRSP sample there are two possible ways to compute quantiles $Q_{i,t}^{MC}$. I report results for the case when each quantile $Q_{i,t}^{MC}$ is calculated on the set of all firms with available market capitalization in the end of year $t$. The results are virtually identical when I define $R_t$ using $Q_{i,t}^{MC}$ and $Q_{i,t-1}^{MC}$ calculated on a subset of firms with observed market capitalization both in years $t$ and $t − 1$, or when the sum in (1) is calculated over the subset of 10% largest firms as of year $t − 1$. 
The first column in Table 1 reports results of the regression of $R_t$ on the innovation activity, output gap and a linear trend. I rescale $IA_t$ and the Output Gap to have unit variance for easier interpretation. The slope coefficient of $IA_t$ is both statistically and economically significant: one standard deviation increase in $IA_t$ corresponds to 0.01 increase in reallocation or about 16% increase relative to its mean value (the mean value is reported in the second to last row). This observation confirms the comovement between the two series that we saw on Figure 1. Inclusion of the output gap in the regression does not have any significant impact on the slope coefficient of $IA_t$ which indicates that the observed comovement is not directly driven by the regular business cycle phenomenon.

The second and third columns of Table 1 report regressions of two additional measures of idiosyncratic risk of public companies on the innovation activity. The first one, denoted by CS StDev, is the cross-sectional standard deviation of annual firm returns. The second, quantile range, is the difference between 95%-tile and 5%-tile of annual returns in the cross-section of firms in a given year. These two regressions confirm the strong positive relationship between idiosyncratic risk and the innovation activity. The slope coefficients for $IA_t$ are positive and significant. Magnitudes of the effect are similar to the case of rank reallocation $R_t$: one standard deviation increase in innovation activity is associated with 18% increase in CS StDev and 15% increase in quantile range relative to the respective means.

Finally, the last column reports regressions of the job reallocation on the innovation activity, output gap and a linear trend. Job reallocation is from the Business Dynamics Statistics. This measure of reallocation covers the set of both private and publicly traded firms in the US. The slope on innovation activity is economically and statistically significant, which indicates that the comovement between idiosyncratic risk and innovation is not restricted to publicly traded firms but is a broad economy phenomenon.

One explanation for this comovement is based on Joseph Schumpeter’s notion of creative destruction. In times of high R&D spending new firms challenge incumbents by attempting to develop new, better quality, goods. Incumbent firms increase R&D spending in search of new ideas to preserve or improve their position in the market space. During this process it is more likely that any given firm will experience a change in its market position, which can either improve or deteriorate. Hence, increased economy-wide innovation activity will be associated with more reallocation between firms.

**Idiosyncratic Risk to Investors’ Wealth** Most businesses, both private and public, have stakeholders for whom the share owned constitutes a large portion of their personal wealth. Owners of privately held firms naturally have limited options for diversification.
Hence value of their undiversified stakes often comprises a large part of their total net worth. Moskowitz and Vissing-Jørgensen (2002) report that households, conditional on owning a stake in a private business, on average have more than 40% of wealth invested in it. In the case of public firms, firm founders or founders’ family members are often heavily invested in the company’s stock and are highly non-diversified. For example heirs of Samuel Walton, the founder of Walmart Inc, own about 50% of Walmart’s common stock. This investment corresponds to more than 80% of their net worth. Apart from founders and families, almost all public firms tie chief executives’ and key partners’ compensation to the firms’ performance. For these employees compensation in the current firm constitutes a large part of their lifetime income.

Given the evidence on firm-level risk presented above, we should expect heavily non-diversified business owners to face higher idiosyncratic risk in times of high innovation activity. Using data on wealth of individuals on the Forbes 400 list I confirm this conjecture.

Since 1982 Forbes magazine has been tracking and publishing on the annual basis the list of 400 wealthiest Americans along with their estimated net worth. Two important features make this data attractive for use in my study in comparison with other few data sources that contain information on personal wealth. First, most individuals stay on the Forbes 400 list for multiple years. This allows me to track individuals in time and calculate returns on their wealth. Second, most members on the Forbes list have non-diversified and, often, active ownership in an operating business, private or public.

I define logarithmic return on wealth according to

\[ r_{i,t}^W = \log \left( \frac{W_{i,t}}{W_{i,t-1}} \right), \]  

where \( W_{i,t} \) is the net worth of individual \( i \) in year \( t \) reported by Forbes.\(^7\)\(^8\) I use logarithmic returns in this setting to avoid the influence of extremely high realizations when calculating measures of idiosyncratic risk. Every year a subset of individuals exits the Forbes 400 list, some of these exits are caused by large drops in wealth of these individuals. Performance related exits from the list introduce a downward bias in the measures of idiosyncratic

\(^6\)As of August 2018 based on Forbes net worth estimates and Walmart Inc 2018 proxy statement.

\(^7\)The ideal definition of return on wealth involves a measure of consumption of individual \( i \), \( C_{i,t} \):

\[ r_{i,t}^W = \log \left( \frac{(C_{i,t} + W_{i,t})}{W_{i,t-1}} \right). \]  

Forbes does not report data on consumption so I define the return on wealth according to equation (2). This simplification should not have any significant impact on the results since the ideal measure perfectly correlates with (2) under the assumption \( C_{i,t} = \text{const} \times W_{i,t} \). Deviations from this condition are small in magnitude compared to fluctuations in wealth.

\(^8\)I use the data collected by Capehart (2014).
Table 2: Innovation activity and portfolio-level idiosyncratic risk. The table presents results of linear regressions of measures $Y_t$ of idiosyncratic risk on innovation activity $IA_t$: $Y_t = \text{Const} + \beta IA_t + \gamma_1 \text{Output Gap}_t + \gamma_2 t + \epsilon_t$. Innovation activity is measured as the detrended ratio of economy-wide R&D/GDP ratio normalized to have zero mean and unit variance. Output gap is normalized to unit variance. CS StDev is the cross-sectional standard deviation of annual returns on wealth, Quantile Range is the difference between 95%-tile and 5%-tile of annual return on wealth, CS StDev$^U$ / CS StDev$^D$ are upside/downside standard deviations of return on wealth. Dependent variables are calculated using the data on 100 wealthiest, as of year $t - 1$, members of the Forbes 400 list. HAC standard errors in parentheses. One, two and three stars denote significance at the 10, 5 and 1% level. Annual observations, 1983–2013.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>CS StDev</th>
<th>Quantile Range</th>
<th>CS StDev$^U$</th>
<th>CS StDev$^D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$IA_t$</td>
<td>0.063***</td>
<td>0.203***</td>
<td>0.041***</td>
<td>0.096**</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.060)</td>
<td>(0.013)</td>
<td>(0.045)</td>
</tr>
<tr>
<td>Output Gap</td>
<td>-0.020*</td>
<td>-0.056</td>
<td>-0.001</td>
<td>-0.043</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.047)</td>
<td>(0.011)</td>
<td>(0.036)</td>
</tr>
<tr>
<td>Mean($Y_t$)</td>
<td>0.270</td>
<td>0.805</td>
<td>0.264</td>
<td>0.279</td>
</tr>
<tr>
<td>R$^2$</td>
<td>0.461</td>
<td>0.448</td>
<td>0.396</td>
<td>0.306</td>
</tr>
</tbody>
</table>

risk. To mitigate this issue I consider the subset of one hundred wealthiest, as of year $t - 1$, individuals from the Forbes 400 list when calculating measures of risk in year $t$. Performance related exits for this subset are extremely rare.\(^9\)

For each year in the sample I calculate two measures of idiosyncratic risk. The first one is the cross-sectional dispersion of the return on wealth. The second measure is the quantile range. As in the case of the firm-level analysis, the quantile range is defined as the difference between 95%-tile and 5%-tile of annual returns on wealth of individuals $r_{i,t}^W$, in year $t$. This definition of idiosyncratic risk is robust to possible outliers but at the same time serves as a good measure of tail risk.

In the first and second column of Table 2 I report the results of OLS regressions of the cross-sectional standard deviation and the quantile range on the innovation activity $IA_t$, output gap and a linear trend. As before, $IA_t$ is the detrended R&D-to-GDP ratio normalized to unit variance. Regression results for both measures are consistent with each other: slope coefficients are statistically significant and positive. The mean value of CS StDev is 0.270 and one standard deviation increase in $IA_t$ corresponds to $0.063/0.270 \approx 23\%$ increase in its value relative to the mean. The mean value of the quantile range is 0.805 and one standard deviation increase in $IA_t$ is associated with

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\(^9\)In the empirical appendix I repeat the exercise for the full sample of individuals on the Forbes 400 list. All conclusions remain unchanged.
about 25% increase in its value relative to the mean. The relationship is both statistically
and economically significant. Quantitatively this pattern is similar to the one observed
for the firm-level risk.

Theoretically, increased intensity of innovation activity can have asymmetric impact
on the left and right tails of the return on wealth distribution. To investigate this question
I define upside and downside versions of idiosyncratic risk measures:

$$CS\ StDev^U_t = \sqrt{\frac{\sum_{i,r} W_{i,t} \geq \mu_t \left( r_{i,t} - \mu_t \right)^2}{N^U}}$$

$$CS\ StDev^D_t = \sqrt{\frac{\sum_{i,r} W_{i,t} < \mu_t \left( r_{i,t} - \mu_t \right)^2}{N^D}}$$

$$\mu_t = \frac{\sum_i r_{i,t} W_{i,t}}{N}$$

where $N^U$ and $N^D$ stand for the number of observations with $r_{i,t} W_{i,t} \geq \mu_t$ and $r_{i,t} W_{i,t} < \mu_t$ correspondingly. I report results of the regressions of these variables on the innovation
activity in columns 3 and 4 of Table 2. On average distribution of log returns on wealth
appears to be not skewed: mean values of measures of upside and downside risk are very
close to each other. Slope coefficients, however, tell us a different story if one conditions
on the amount of innovation activity. Distribution of returns is skewed relatively more
to the left when innovation activity is high. I can not, however, statistically reject the
null hypothesis that slope coefficients are equal for upside and downside risk against
the one sided alternative that downside risk slope is higher than upside at the 5% level.
The p-value of such test is equal to 0.12 (not reported in the table). This exercise thus
provides mild evidence that owners of incumbent businesses face higher left tail risks
when the economy undergoes high innovation activity periods.

**Aggregate Market Volatility** The first column of Table 3 reports results of regressions of realized volatility of the CRSP value-weighted returns on innovation activity,
output gap and linear trend. The slope coefficient on $IA_t$ is not significant and economi-
cally small. Columns two through four investigate potential conovent of lagged
innovation activity and market volatility. In regressions with lags all slope coefficients
on innovation activity are positive and are either insignificant or marginally significant.
There is only weak positive correlation in innovation activity and stock market volatility.

From the standpoint of an undiversified investor these stylized facts suggest relative
importance of the redistributive effects of innovation. The model formulated in the next
Table 3: **Innovation activity and market portfolio volatility.** The table presents results of linear regressions of measures the market portfolio annual realized volatility $RV_t$ on innovation activity $IA_t$: $RV_t = Const + \beta IA_t + \gamma_1$Output Gap $+ \gamma_2 t + \epsilon_t$. Innovation activity is measured as the detrended ratio of economy-wide R&D/GDP ratio normalized to have zero mean and unit variance. Output gap is normalized to unit variance. Realized volatility is defined as the square root of the annual realized variance of CRSP value-weighted returns calculated using daily data. HAC standard errors in parentheses. One, two and three stars denote significance at the 10, 5 and 1% level. Annual observations, 1953–2016.

<table>
<thead>
<tr>
<th></th>
<th>Dependent variable:</th>
<th>Market Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>$IA_t$</td>
<td></td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.010)</td>
</tr>
<tr>
<td>$IA_{t-1}$</td>
<td></td>
<td>0.013*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.007)</td>
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<td>$IA_{t-2}$</td>
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<td></td>
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<td>(0.008)</td>
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<td>$IA_{t-3}$</td>
<td></td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.009)</td>
</tr>
<tr>
<td>Output Gap</td>
<td>$-0.006$</td>
<td>$-0.006$</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.007)</td>
</tr>
<tr>
<td></td>
<td>$-0.006$</td>
<td>$-0.006$</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
</tr>
<tr>
<td></td>
<td>$-0.007$</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Mean($Y_t$)</td>
<td>0.137</td>
<td>0.137</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>R$^2$</td>
<td>0.234</td>
<td>0.260</td>
</tr>
<tr>
<td></td>
<td>0.241</td>
<td>0.217</td>
</tr>
</tbody>
</table>

section captures these observations.

# 3 Model Setup

## 3.1 Firms

The economy is populated by a set of firms operating in a variety of product lines. Overall there is a continuum of measure one of product lines indexed by $i$, $i \in [0, 1]$. Each product line is characterized by the latest technology or the latest quality $q_{i,t}$ at time $t$, $q_{i,t} > 0$. At any point in time there is exactly one incumbent firm in a given product line $i$. The incumbent firm owns production rights on the good of the highest quality within its product line.

Each incumbent firm generates a flow operating profit of

$$\Pi_{i,t} = a \times q_{i,t}. \quad (6)$$
Here $a$ is a positive constant which I normalize to be equal to one. This setup emerges as an outcome of monopolistic competition in a class of quality ladder type models of growth (Acemoglu, 2008).

Operating profit $\Pi_{i,t}$ can be either distributed to the firm’s shareholders or invested in innovation inside the firm. The quality of variety $i$ evolves according to

$$\frac{dq_{i,t}}{q_{i,t}} = (g(\iota_{i,t}) + y_t) \, dt + (\lambda - 1) dJ_{i,t} + \sigma dB_t. \quad (7)$$

Here $g(\iota_{i,t})$ is expected growth through internal incremental innovation given the flow of investment per unit of quality $\iota_{i,t}$. Function $g(\cdot)$ is deterministic, increasing and concave. Term $y_t$ is a small, mean-zero and persistent component of expected growth

$$dy_t = -\kappa_y y_t dt + \sigma_y^y dB^y_t, \quad (8)$$

where $dB^y_t$ is an exogenous Brownian shock. Exogenous growth $y_t$ in a reduced form proxies for time-varying economic conditions that are important for firms’ and agents’ policies but are not modeled explicitly. I follow Bansal and Yaron (2004) to calibrate parameters $\kappa_y$ and $\sigma_y^y$ in my quantitative analysis.

The second term in equation (7), $(\lambda - 1)dJ_{i,t}$, is the central element of the dynamics of quality $q_{i,t}$ and the main focus of the model. It represents the process of radical or disruptive innovation in variety $i$. The symbol $J_{i,t}$ denotes the Poisson counting process, $J_{i,t} \in \mathbb{N}_0$. Constant $\lambda$ denotes a proportional quality increase in a given variety conditional on a successful radical innovation, $\lambda > 1$. In the next section I describe the process of radical innovation and the structure of shock $dJ_{i,t}$ in more detail. The term $dB_t$ is the aggregate Brownian shock. This shock uniformly affects productivity of all firms in the economy. All three aggregate shocks, $dB^y_t$, $dJ_t$, and $dB_t$ are not correlated. This model does not feature any fully exogenous idiosyncratic shocks (although it is straightforward to add them). Innovation shock $dJ_{i,t}$ is the sole source of idiosyncratic risk that I am focusing on in this paper. Equation (7) can be rewritten in the integrated form as

$$q_{i,t} = q_{i,0} \times e^\int_0^t (g(\iota_{i,u}) + y_u) \, du \times \lambda^{J_{i,t}} \times e^{\sigma B_t - \frac{\sigma^2}{2} t}. \quad (9)$$

According to this formula the current quality of variety $i$ is the result of accumulation of (i) gradual improvements, (ii) radical improvements and (iii) aggregate productivity shocks.
3.2 Radical Innovation and the Innovation Sector

Radical innovation is the source of creative destruction in the economy. This type of innovation can be pursued both by incumbent firms and by new entrants. Active firms invest in radical innovation to improve their profitability and protect their current business from a potential disruption. The goal of an entrant is to achieve a radical improvement in one of the varieties and displace the incumbent.

Entrants’ R&D efforts are fully financed by the innovation sector. The innovation sector is a financial intermediary that accepts contributions from outside investors and allocates the raised funds optimally across new business ventures. I interpret the innovation sector very broadly and its real world counterpart would include the majority of the sources through which new businesses attract outside money. Among others, these sources include venture capital, angel investing, equity crowdfunding and initial public offerings of young high-growth companies.

Radical innovations happen in aggregate waves. Each wave comes with a constant and exogenous Poisson intensity $\bar{h}$. I denote by $J_t$ the Poisson counting process that governs their arrival. Conditional on arrival of a wave at time $t$, $dJ_t = 1$, three distinct scenarios are possible for a given variety $i$:

1. With probability $\omega^I_{i,t}$ the incumbent firm achieves radical improvement of the product line it owns, so $q_{i,t} = \lambda q_{i,t-} - dJ_{i,t} = 1$.

2. With probability $\omega^E_{i,t}$ a new firm succeeds in radical innovation, replacing the incumbent and starting production of variety $i$ of quality $q_{i,t} = \lambda q_{i,t-} - dJ_{i,t} = 1$.

3. With probability $(1 - \omega^I_{i,t} - \omega^E_{i,t})$ neither the incumbent nor the entrant succeed in radical innovation; the incumbent keeps producing variety $i$ of quality $q_{i,t} = q_{i,t-}$ and $dJ_{i,t} = 0$.

Conditional on $dJ_t = 1$, realizations of these scenarios are independent across $i$. Probabilities $\omega^I_{i,t}$ and $\omega^E_{i,t}$ are determined by the flows of investment into radical innovation made by the incumbent $z^I_{i,t}$ and allocated by the innovation sector $z^E_{i,t}$ to variety $i$, according
Parameters in the above equations satisfy the constraints: $\chi^E, \chi^I > 0, \alpha^E, \alpha^I \in (0, 1), s \in [0, 1]$. For fixed $t$ optimal investment decisions in this setup are homothetic in $q_{i,t}$ across firms. Hence for brevity, I introduce additional notation $\hat{z}^E_{i,t} = z^E_{i,t}/q_{i,t}$, $\hat{z}^I_{i,t} = z^I_{i,t}/q_{i,t}$, and in many cases omit subscripts $i$ and $t$.

The functional specification of the radical innovation technology has several important properties. First, the parametric form in equations (10a–10b) satisfies the Inada conditions. Second, it insures that, conditional on arrival of a wave, probability of successful innovation by the incumbent or an entrant does not exceed one. Equations (10c–10d) insure that total conditional probability of innovation by the incumbent and an entrant does not exceed one $\omega^E_{i,t} + \omega^I_{i,t} \leq 1$. In addition, the last two equations capture the notion of competition between incumbents and entrants in the technological race. Incumbent’s (entrant’s) probability of success in radical innovation decreases with the amount of investment made by the innovation sector (the incumbent). I assume that, when making a choice of $z^I_{i,t}$, the incumbent firm takes investment flow from the innovation sector $z^E_{i,t}$ as given. The investment flow into the innovation sector is optimally allocated between different varieties to maximize the return on investment, taking innovation intensity by incumbents $z^I_{i,t}$ as given.

The assumption of lumpy arrival of radical innovations in a novel way unites the models by Aghion and Howitt (1992) and Grossman and Helpman (1991). It allows the technological progress to have heterogeneous impact on firms and at the same time allows me to study the risk premium on innovation. Pástor and Veronesi (2009), Garleanu et al. (2012) develop models with aggregate technological waves and asset pricing effects as well, but focus on the process of learning and adoption of new technologies correspondingly.
3.3 Agents and Capital Markets

The left panel of Figure 2 summarizes the structure of the model. The economy is populated by capitalists who can be of two types: households and managers. The type of each agent is not permanent and can change with the arrival of an innovation wave as will be described below. Households consume out of their wealth and save through investments in the stock market (operating firms), the risk-free asset, the innovation sector and an additional technical security, which I call security-\(y\). By definition, return on security-\(y\) is perfectly correlated with shock \(dB^y_t\). This setup insures that financial markets are dynamically complete with respect to the three aggregate shocks \(dB^y_t\), \(dJ_t\) and \(dB_t\).

The risk-free asset and security-\(y\) are in zero net supply.

As households, managers consume out of their savings and have access to the same investible securities. Crucially, there is a one-to-one mapping between managers and incumbent firms. The manager matched to firm \(i\) makes dividend payout and investment decisions \(\nu_{i,t}\) and \(z_{i,t}^I\) on behalf of the firm. On top of that, the manager is forced to keep at least share \(\phi\) of her net worth invested in the firm’s stock, \(\phi \geq 0\). Parameter \(\phi\) in a reduced form captures necessary incentive provisions for the manager. This type of constraint on manager’s ownership can be rationalized in the setup with moral hazard and multiplicative utility, akin to Edmans et al. (2008). Managers in my model should be interpreted broadly and include individuals who have non-diversified ownership of an operating business. These include business founders, family owners, company officers, board members and other kinds of non-diversified individual investors.

The assumption that managers and households invest their liquid wealth in the broad

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\(^{10}\)Presence of security-\(y\) makes the model more tractable but is not essential for the results.
stock market index, as opposed to a subset of stocks, is without loss of generality. One could allow agents to invest in single stocks, but due to the risk aversion each agent will pick the fully diversified stock market index. When investing in the innovation sector, however, neither managers nor households have control over the allocation of their investments into the exact varieties. In cross-section, payouts to investors made by the innovation sector are returned in proportion to investors’ original contributions.

For tractability I assume that households constitute a double-continuum set, so that for every firm or manager in the economy there is a continuum of households. Collectively aggregate wealth of households is comparable to the aggregate wealth of managers. Each manager, however, is substantially more wealthy than a typical household $w^M \gg w^H$. This setup can be thought of as an economy with $N$ firms, $N$ managers and $\sim N^2$ households in the limiting case $N \to +\infty$. This technical assumption allows characterizing the evolution of the aggregate state of the economy without keeping track of complete distribution of wealth among agents.

Both managers and households have stochastic differential utility introduced by Duffie and Epstein (1992):

$$U_t = \mathbb{E}_t \int_t^{\infty} f(c_u, U_u) du,$$

(11)

$$f(c_u, U_u) = \frac{\rho}{1 - \psi} \left( \frac{c^{1-\psi}((1 - \gamma)U)^{\frac{\gamma}{1-\gamma}}}{(1 - \gamma)U} \right).$$

(12)

The right panel of Figure 2 illustrates the arrival of an innovation wave $dJ_t = 1$. Upon arrival of the wave share $\int \omega^I_i di$ of firms succeeds in radical innovation and improves the quality of the product by a factor of $\lambda$. Share $\int \omega^E_i di$ of firms gets disrupted by new entrants who improve the quality of corresponding products by $\lambda$ as well. Disrupted firms exit the market and corresponding managers become households and consume out of their remaining net worth. Each entering firm is managed by a new manager chosen randomly from the set of households. The new manager receives share $\beta$ of the firm value. Share $(1 - \beta)$ of the entering firm value constitutes part of the innovation sector’s profits and is paid back to investors.

\[11\]\footnote{A continuous time extension of the recursive preferences studied by Kreps and Porteus (1978), Epstein and Zin (1989), Weil (1990).}
4 Solving the Model

The economy features three aggregate shocks: Brownian shocks $dB_t$ and $dB^y_t$ and the jump shock $dJ_t$. The first two shocks affect uniformly all firms in the economy; the last benefits some firms but hurts the other. Financial markets are complete with respect to aggregate shocks, hence the stochastic discount factor $\eta_t$ can be written as a jump-diffusion process

$$\frac{d\eta_t}{\eta_t} = -r_t \, dt - \pi_t \, dB_t - \pi^y_t \, dB^y_t - \pi^J_t \, (dJ_t - \bar{h} \, dt). \quad (13)$$

Here $r_t$ is the risk-free rate, $\pi_t$ and $\pi^y_t$ are the market prices of risk to aggregate productivity and expected growth, $\pi^J_t$ is the market price of jump (or innovation wave) risk. The price of jump risk has the following economic interpretation: an agent willing to receive a payoff of $\Delta$ in case of jump arrival $dJ_t = 1$ has to pay a flow $\Delta \times \bar{h}(1 - \pi^J_t)$ in exchange, where $\bar{h}$ is the intensity of jump arrival. When the price of jump risk is equal to zero, $\pi^J_t = 0$, the money flow paid by the agent in an instant of time $\Delta \times \bar{h}(1 - \pi^J_t) \, dt$ is equal to the expected payoff $\Delta \times \bar{h} \, dt$. When the price of jump risk is positive (negative) the money flow paid in an instant of time is lower (higher) than the expected payoff.

In a symmetric equilibrium all operating firms will have the same stock price per unit of quality $p_t$. I postulate the process for $p_t$

$$\frac{dp_t}{p_t} = \mu_{p,t} \, dt + \sigma_{p,t} \, dB_t + \sigma^y_{p,t} \, dB^y_t + \sigma^J_{p,t} \, dJ_t. \quad (14)$$

4.1 Problem of the Innovation Sector

The innovation sector maximizes the payoff to the investors conditional on wave arrival

$$\max_{z_{i,t}^E} \lambda(1 - \beta)p_t \left(1 + \sigma^J_{p,t}\right) \int q_{i,t} \omega_{i,t} \left(z_{i,t}^{E}, z_{i,t}^{I}\right) \, di \quad (15)$$

subject to the constraint

$$\int z_{i,t}^E \, di = Z_{i}^E, \quad (16)$$

where by $Z_{i}^E$ I denote the aggregate investment flow into the innovation sector. The term $\lambda(1 - \beta)p_t(1 + \sigma_{p,t})$ in equation (15) does not impact the allocation decision, but I include it for clarity. The innovation sector takes as given the aggregate inflow $Z_{i}^E$ and investment in radical innovation by incumbent firms $z_{i,t}^I$. The inflow $Z_{i}^E$ is determined by optimal portfolio choices of households and managers.
4.2 Problem of Households

Households choose optimal consumption and optimal investments in the risk-free asset, the stock market and the innovation sector. Since available investment opportunities span all aggregate shocks, the households’ portfolio choice problem is equivalent to the choice of exposure to these shocks. Households choose optimal consumption $c^H_t$, optimal exposures to the aggregate productivity shock $\sigma^{w,t}_H$ and shock to the expected growth $\sigma^{y,t}_H$, and finally exposure to the jump risk $\sigma^{J,H}_t$ to solve

$$
\max_{c^H_t \geq 0, \sigma^{w,t}_H, \sigma^{y,t}_H, \sigma^{J,H}_t} \mathbb{E}_t \int_t^\infty f(c^H_u, U^H_u) \, du
$$

subject to the budget constraint

$$
\frac{dw^H_t}{w^H_t} = -\dot{c}^H_t \, dt + \left( r_t + \sigma^{w,t}_H \pi_t + \sigma^{y,t}_H \pi^y_t + \sigma^{J,H}_t (\pi^J_t - 1) \bar{h} \right) \, dt + \sigma^{H}_w \, dB^H_t + \sigma^{y,H}_w \, dB^y_t + \sigma^{J,H}_w \, dJ_t.
$$

(18)

Here $w^H_t$ is agent’s wealth, $\dot{c}^H_t$ is her consumption normalized by wealth. In equilibrium all households will choose the same optimal investment portfolio and consumption rate.

4.3 Problem of Managers

Managers choose consumption $c^M_t$ and solve the portfolio allocation problem by choosing exposures $\{\tilde{\sigma}^{M}_{w,t}, \tilde{\sigma}^{y,M}_{t,w}, \tilde{\sigma}^{J,M}_{t,w}\}$ of their liquid wealth to aggregate shocks. Tildes in the notation indicate association with the liquid share of wealth. In addition, managers make decisions on the investment policies $\{i_{i,t}, \tilde{z}^{I}_{i,t}\}$ of the firms they run. Hence the problem of a manager is written as

$$
\max_{c^M_t \geq 0, \tilde{\sigma}^{M}_{w,t}, \tilde{\sigma}^{y,M}_{t,w}, \tilde{\sigma}^{J,M}_{t,w}, i_{i,t}, \tilde{z}^{I}_{i,t}} \mathbb{E}_t \int_t^\infty f(c^M_u, U^M_u) \, du
$$

subject to the budget constraint

$$
\frac{dw^M_t}{w^M_t} = -\dot{c}^M_t \, dt + \phi \, dR_{i,t} + (1 - \phi) \, d\tilde{R}_t,
$$

(20)
where \( dR_{i,t} \) is return on the stock of the firm she controls, and \( \tilde{R}_t \) is return on the liquid part of her wealth. In equation (20) I implicitly take into account the fact that a risk averse manager chooses minimal possible allocation of her wealth in her own firm \( \phi \). As in the case of households, the return on liquid wealth is defined by

\[
d\tilde{R}_t = \left( r_t + \tilde{\sigma}^M_{w,t} \pi_t + \tilde{\sigma}^{yM}_{w,t} \pi^{y}_t + \tilde{\sigma}^{JM}_{w,t} (\pi^{J}_t - 1) \tilde{h} \right) dt
\]

\[+ \tilde{\sigma}^M_{w,t} dB_t + \tilde{\sigma}^{yM}_{w,t} dB^{y}_t + \tilde{\sigma}^{JM}_{w,t} dJ_t. \tag{21}\]

The return on firm’s \( i \) stock consists of the dividend yield and capital gains

\[
dR_{i,t} = \left( a - \iota_{i,t} - \tilde{z}^I_{i,t} \right) \frac{dt}{p_t} + \frac{d(p_t q_{i,t})}{p_t q_{i,t}} , \tag{22}\]

where I took into account that firm’s \( i \) dividend is equal to the difference between the operating profit \( a q_{i,t} \) and total spending on gradual and radical innovation \( q_{i,t} (\iota_{i,t} + \tilde{z}^I_{i,t}) \).

Applying Ito’s lemma to the second term in the equation above, and taking into account dynamics of \( q_{i,t} \) and \( p_t \) defined in equations (7) and (14), I obtain

\[
dR_{i,t} = \left( a - \iota_{i,t} - \tilde{z}^I_{i,t} \right) \frac{dt}{p_t} + \frac{d(p_t q_{i,t})}{p_t q_{i,t}} + \frac{\sigma p_t dJ_{i,t} + \sigma yM_{w,t} dB_{y, t} + (1 - \phi) \sigma JM_{w,t} dJ_E}{\phi (1 + \sigma_{p,t} + \sigma_{p,t} - dJ_{i,t} - dJ_{E,i,t} - dJ_{E,i,-}) - dJ_{i,t} + \lambda (1 + \sigma_{p,t} - 1) dJ_{i,t}} , \tag{23}\]

where by \( dJ_{i,t} \) and \( dJ_{E,i,t} \) I denote successful radical innovations in variety \( i \) by the incumbent and an entrant correspondingly. By substituting definitions of \( d\tilde{R}_t \) and \( dR_{i,t} \) we can rewrite the budget constraint (20) as

\[
\frac{dw^M_{i,t}}{w^M_{i,t}} = -\tilde{c}^M_t dt + \mu^M_{w,t} + \sigma^M_{w,t} dB_t + \sigma^{yM}_{w,t} dB^{y}_t + (1 - \phi) \sigma^{JM}_{w,t} dJ_t
\]

\[+ \phi \left( \sigma_{p,t} - (dJ_t - dJ_{i,t} - dJ_{E,i,t}) - dJ_{E,i,t} + \lambda (1 + \sigma_{p,t} - 1) dJ_{i,t} \right) , \tag{24}\]

where I introduced additional notation

\[
\mu^M_{w,t} = (1 - \phi) \mu_{\tilde{R},t} + \phi \mu_{R,t} , \tag{25}\]

\[
\sigma^M_{w,t} = (1 - \phi) \tilde{\sigma}^M_{w,t} + \phi (\sigma + \sigma_{p,t}) , \tag{26}\]

\[
\sigma^{yM}_{w,t} = (1 - \phi) \tilde{\sigma}^{yM}_{w,t} + \phi \sigma_{p,t} , \tag{27}\]

20
and $\mu_{R,t}$ and $\mu_{\tilde{R},t}$ are drifts of the continuous components of $d\tilde{R}_t$ and $dR_t$ given by (see equations 21, 23)

\[
\begin{align*}
\mu_{\tilde{R},t} &= r_t + \tilde{\sigma}_{\tilde{R},w,t} \pi_t + \tilde{\sigma}_{\tilde{R},y,M} \pi^y_{w,t} + \tilde{\sigma}_{\tilde{R},J,M} (\pi^J_t - 1) \bar{h}, \\
\mu_{R,t} &= a - \iota(g_{i,t}) - \hat{z}_{i,t}^I + g_{i,t} + \mu_{p,t} + \sigma_{p,t}.
\end{align*}
\] (28)

**Equilibrium**

The definition of equilibrium involves aggregation of policies and other equilibrium variables across households, managers, and productive firms. Hence I introduce notation $\int_H f_j \, dj$ and $\int_M f_i \, di$ to represent a quantity $f$ aggregated across the set of households and managers correspondingly. Since there is a one-to-one mapping between managers and product lines, $\int_M f_i \, di$ also denotes the value of $f$ aggregated across product lines.

**Definition 1.** An equilibrium is a set of stochastic processes for the price per unit of quality $\{p_t\}$, the stochastic discount factor $\{\eta_t\}$; for each product line $i$ the processes for quality $\{q_{i,t}\}$, optimal investment policies $\{\iota_{i,t}, z_{i,t}^I, z_{i,t}^E\}$ by incumbents and the innovation sector; for each manager $i$ and household $j$ wealth $\{w^M_{i,t}, w^H_{j,t}\}$, portfolio loadings $\{\tilde{\sigma}_{i,w,t}^M, \tilde{\sigma}_{i,w,t}^y, \tilde{\sigma}_{j,w,t}^J, \sigma_{j,w,t}^y\}$ and consumption $\{c^M_{i,t}, c^H_{j,t}\}$ such that:

1. Taking aggregate conditions and investment of the innovation sector $z_{i,t}^E$ as given households and managers maximize their respective utilities subject to their budget constraints,

2. Taking investment in radical innovation by incumbents $z_{i,t}^I$ as given the innovation sector solves its optimal allocation problem,

3. Market for consumption goods clears

\[
\int_M c^M_{i,t} \, di + \int_H c^H_{j,t} \, dj = \int_M ((1 - \iota_{i,t})q_{i,t} - z_{i,t}^I) \, di - \int_M z_{i,t}^E \, di,
\] (30)

4. Financial markets clear

\[
(1 - \phi) \int_M \tilde{\sigma}_{i,w,t}^M w^M_{i,t} \, di + \int_H \sigma_{j,w,t}^H w^H_{j,t} \, dj = (\sigma + \sigma_{p,t}) \int_M (p_t q_{i,t} - \phi w^M_{i,t}) \, di,
\] (31)

\[
(1 - \phi) \int_M \tilde{\sigma}_{i,w,t}^y w^M_{i,t} \, di + \int_H \sigma_{j,w,t}^y w^H_{j,t} \, dj = \sigma_{p,t} \int_M (p_t q_{i,t} - \phi w^M_{i,t}) \, di,
\] (32)
\[
(1 - \phi) \int_M \hat{\sigma}_{i,w,t}^J w_i^M dt + \int_H \sigma_{i,w,t}^J w_i^H dt = \\
(1 - \beta) \lambda (1 + \sigma_{p,t}) \int_M p_t q_{i,t} \omega_{E}^{i,t} dt \\
+ \sigma_{p,t} \int_M (1 - \omega_{E}^{i,t}) (p_t q_{i,t} - \phi w_{i,t}^M) dt \\
+ (\lambda(1 + \sigma_{p,t}^J) - 1) \int_M \omega_{E}^{i,t} (p_t q_{i,t} - \phi w_{i,t}^M) dt \\
- \int_M \omega_{E}^{i,t} (p_t q_{i,t} - \phi w_{i,t}^M) dt.
\] (33)

In the definition above conditions 1–3 are standard. Condition 4 and equations (31, 32, 33) define market clearing for the aggregate productivity, growth and jump risks correspondingly. The market for the risk-free asset clears by Walras’ law. The left-hand side of each of the three equations in condition 4 corresponds to the aggregate demand for a given type of risk, the right-hand side — to aggregate supply. For each type of risk aggregate demand has the same structure and consists of two parts: aggregated across managers exposure of liquid wealth and aggregated exposure of households’ wealth. Aggregate supply of the exposure to shock \( dB_t \) (right-hand side of equation 31) is equal to the product of the aggregate stock market return exposure to shock \( dB_t \), \( (\sigma + \sigma_{p,t}) \), times the value of the free-float equity \( \int (p_t q_{i,t} - \phi w_{i,t}^M) dt \). The free-float is defined as the aggregate value of the stock market excluding the shares that managers have to keep in their own firms. Supply of exposure to shock \( dB_t \) has the same structure. Supply of exposure to the jump risk consists of four parts. The first line on the right-hand side of equation (33) calculates the total payoff to the investors in the innovation sector conditional on wave arrival. The second line aggregates the change in value of firms that are not directly affected by the innovation wave. The third line accounts for the firms that succeed in radical innovation. The fourth accounts for the firms that get displaced conditional on an arrival of a wave.

4.4 Solution

The general solution strategy follows Di Tella (2017). The value function of a household has the form
\[
U^H_t (w^H_t) = \dfrac{(\xi^H_t w^H_t)^{1-\gamma}}{1-\gamma},
\] (34)

where \( \xi^H_t \) is a stochastic process that reflects household’s investment opportunities. It depends on the aggregate state of the economy and does not depend on wealth \( w^H_t \).
Homogeneity of the value function $U^H_t(w^H_t)$ in wealth follows from three observations. First, preferences defined in (11) are homogeneous in consumption of degree $(1 - \gamma)$. Second, households’ budget constraint (18) is linear in wealth. Third, probability for a household to become a manager conditional on wave arrival is infinitesimal.

I focus on the equilibrium with symmetric investment policies pursued by managers. In such equilibrium investment in gradual growth $\iota_{i,t}$, investment in radical innovation normalized by quality $\hat{z}_{i,t}$ by incumbent, and $\hat{z}_{i,t}^E$ by the innovation sector do not depend on $i$. Probability of each manager to become a household conditional on a wave arrival $\omega_{i,t}^E$ does not depend on $i$ as well. Hence homogeneity of preferences and households’ value function (equation 34) imply that the value function of a manager is homogeneous in wealth as well

$$U^M_t(w^M_t) = \left(\xi^M_t w^M_t\right)^{1-\gamma}$$

(35)

Processes for $\xi^H_t$ and $\xi^M_t$ follow jump-diffusions

$$\frac{d\xi^i_t}{\xi^i_t} = \mu^i_{\xi,t} dt + \sigma^i_{\xi,t} dB_t + \sigma^y_{\xi,t} dB^y_t + \sigma^J_{\xi,t} dJ_t, \quad i \in \{H, M\},$$

(36)

where $\mu^i_{\xi,t}$, $\sigma^i_{\xi,t}$, $\sigma^y_{\xi,t}$, $\sigma^J_{\xi,t}$ are to be determined in equilibrium. Appendix derives the HJB equations for households and managers. The following lemma formulates the optimal policies (except for optimal loadings on jump risk and investment in radical innovation which I describe in the next paragraph):

**Lemma 1.** Optimal consumption policies of households and managers are given by

$$\hat{c}^i_t = \rho^\frac{1}{\psi} \left(\xi^i_t\right)^{\frac{\psi - 1}{\psi}}, \quad i \in \{H, M\}.$$  

(37)

Optimal loadings on aggregate productivity shock $dB_t$ and shock to exogenous growth $dB^y_t$ are given by

$$\sigma^i_{w,t} = \frac{\pi_t + (1 - \gamma)\sigma^i_{\xi,t}}{\gamma}, \quad \sigma^y_{w,t} = \frac{\pi_t + (1 - \gamma)\sigma^y_{\xi,t}}{\gamma}, \quad i \in \{H, M\}.$$  

(38)

Investment in gradual innovation $\iota_t$ is given by

$$g'(\iota_t) = 1/p_t.$$  

(39)

**Radical Innovation and Portfolio Loadings on Jump** Now I turn to investigating agents’ optimal loadings on the jump risk. Households choose optimal exposure to jump
risk by solving the problem

\[
\max_{\sigma_{w,t}^{J,H}} \left\{ \sigma_{w,t}^{J,H} (\pi_{t}^{J} - 1) + \frac{1}{1 - \gamma} \times \left[ \left( \left( \sigma_{\xi,t}^{J,H} \right)^{1 - \gamma} - 1 \right) \right] \right\}.
\] (40)

In absence of arbitrage the price of jump risk \(\pi_{t}^{J}\) is less than one. When choosing \(\sigma_{w,t}^{J,H}\) households weight the tradeoff between the change in wealth \(\sigma_{w,t}^{J,H} (\pi_{t}^{J} - 1)\) per unit of time and change in utility conditional on wave arrival captured by the second term in equation (40). The utility change depends on the jump in investment opportunities \(\sigma_{\xi,t}^{J,H}\) and a proportional change in wealth \(\sigma_{w,t}^{J,H}\) chosen by the agent. The first order condition to problem (40) determines the optimal loading on jump risk \(\sigma_{w,t}^{J,H}\)

\[
\sigma_{w,t}^{J,H} = \left( 1 + \sigma_{\xi,t}^{J,H} \right)^{1 - \gamma} (1 - \pi_{t}^{J})^{-\frac{1}{\gamma}} - 1.
\] (41)

The managers’ problem is more complex,

\[
\max_{\tilde{\sigma}_{w,t}^{J,M}, \hat{z}_{t}^{I}} \left\{ -\frac{\hat{\alpha}_{t}^{I}}{p_{t}} + (1 - \phi) \tilde{\sigma}_{w,t}^{J,M} (\pi_{t}^{J} - 1) \right\}
\]

\[
\begin{align*}
&+ \frac{\hat{h}}{1 - \gamma} \times \left[ \left( \left( \sigma_{\xi,t}^{J,M} \right)^{1 - \gamma} - 1 \right) \right] \\
&+ \frac{\hat{h} \hat{w}_{t}^{I}}{1 - \gamma} \times \left[ \left( \left( \sigma_{\xi,t}^{J,M} \right)^{1 - \gamma} - 1 \right) \right] \\
&+ \frac{\hat{h} \hat{w}_{t}^{E}}{1 - \gamma} \times \left[ \left( \sigma_{\xi,t}^{J,M} \right)^{1 - \gamma} - 1 \right] \right\}.
\] (42)

where probabilities \(\hat{w}_{t}^{I}\) and \(\hat{w}_{t}^{E}\) depend on \(\hat{z}_{t}^{I}\) and \(\hat{z}_{t}^{E}\) according to equations (10a–10d).

A manager jointly chooses the optimal exposure of her liquid wealth to jump \(\tilde{\sigma}_{w,t}^{J,M}\) and spending on radical innovation inside the firm \(\hat{z}_{t}^{I}\). The first line in equation (42) accounts for the decrease in dividend due to investment in radical innovation and continuous change in wealth due to exposure to jump of the liquid wealth. Lines two through four account for the three alternatives that might happen upon wave arrival. First, conditional on a wave arrival with probability \((1 - \hat{w}_{t}^{I} - \hat{w}_{t}^{E})\) the managed firm is not affected by the innovation wave. The manager’s investment opportunities jump by a factor of \(\sigma_{\xi,t}^{J,M}\) and the firm’s market capitalization changes by a factor of \(\sigma_{p,t}^{J,I}\) due to the jump in the stock.

\[\text{12This equation follows from collecting the terms that depend on } \sigma_{w,t}^{J,H} \text{ in the households’ HJB. See appendix.}\]

\[\text{13This equation follows from collecting the terms that depend on } \sigma_{w,t}^{J,M} \text{ and } \hat{z}_{t}^{I} \text{ in the managers’ HJB. See appendix.}\]
price per unit of quality. Second, with probability $\omega_I$ the firm succeeds in the radical innovation and its market capitalization jumps up by $\lambda(1 + \sigma_{I,t})$. Third, with probability $\omega_E$ the managed firm is displaced by an entrant. As a result, the manager loses share $\phi$ of her wealth. In addition, the manager joins the set of households and investment opportunities change by a factor of $(\xi_t^H / \xi_t^M)(1 + \sigma_{J,H}^{tM})$.

Financial loss associated with disruption is the highest threat that a manager faces. She can reduce the conditional probability $\omega_E$ of this outcome by investing more in radical innovation $\hat{z}_I$, but she can’t drive it all the way to zero: as a function of $\hat{z}_I$, probability of disruption $\omega_E$ satisfies the Inada conditions for any value of $\hat{z}_I$. Hence, the manager will tend choose higher loadings on jump $\hat{\sigma}_{J,M}^t$ to hedge against a potential financial loss.

**Equilibrium Inflow into the Innovation Sector** Aggregate flow into the innovation sector $Z_t^E$ equals $\hat{z}_t^E \times Q_t$, where $Q_t$ is the aggregated quality of goods in the economy $Q_t = \int q_{i,t} \text{d}i$. The quantity $\hat{z}_t^E$ is determined from the absence of arbitrage condition

$$\hat{z}_t^E Q_t = \left(1 - \beta\right) \lambda p_t \left(1 + \sigma_{p,t}^J\right) \frac{\omega^E(\hat{z}_t^E, \hat{z}_t^I)}{\hat{\nu}_{t}} \times \left(1 - \pi_t^I\right). \quad (43)$$

The right-hand side of this equation consists of two components. The first one corresponds to the expected payoff to investors. It is equal to the share of new firms going to investors $(1 - \beta)$, times the value of new firms $\lambda p_t \left(1 + \sigma_{p,t}^J\right)$, times intensity of innovation arrival $\hat{\nu}_t$. The second part is the adjustment for the jump risk $(1 - \pi_t^I)$. When the jump risk is not priced, $\pi_t^I \equiv 0$, equation (43) coincides with the free-entry in innovation condition, common to models of endogenous growth.

Rearranging equation (43) we obtain

$$\frac{\hat{z}_t^E}{\omega^E(\hat{z}_t^E, \hat{z}_t^I)} = (1 - \beta) \lambda p_t \left(1 + \sigma_{p,t}^J\right) \frac{\hat{\nu}_t}{\hat{\nu}_{t}} \times \left(1 - \pi_t^I\right). \quad (44)$$

The left-hand side of (44) is an increasing function of $\hat{z}_t^E$ taking investment in radical innovation by the incumbent $\hat{z}_t^I$ fixed.\(^{14}\) Hence, investment in radical innovation by entrants $\hat{z}_t^E$ is increasing in the price of stock conditional on success $p_t \left(1 + \sigma_{p,t}^J\right)$ and

\(^{14}\)Indeed, denote $g(z) = \omega^E(z, \hat{z}_t^I)$. Function $g(z)$ is strictly concave and $g(0) = 0$, see equation (10c). Take any two points $z_1$ and $z_2$, so that $z_2 > z_1 > 0$. Then $g(z_2)/z_2 - g(z_1)/z_1 = ([g(z_2) \times z_1/z_2 + g(0) \times (z_2 - z_1)/z_2 - g(z_1)]/z_1$. The nominator of the last expression is less than zero by the definition of a strictly concave function, hence $g(z_2)/z_2 < g(z_1)/z_1$ or equivalently $z_2/g(z_2) > z_1/g(z_1)$. 

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decreasing in the price of jump risk $\pi_t^J$. The price of jump risk becomes an important determinant of the amount of radical innovation.

4.5 State Variables

I focus on equilibrium that is Markov in three state variables: (i) exogenous growth component $y_t$, (ii) scale of the economy as measured by the average quality $Q_t$, (iii) share of wealth owned by the managers $x_t$. I don’t need to keep track of the distribution of wealth within each class of agents and distribution of qualities across product lines due to homogeneity of policy functions.

Share of wealth owned by the managers $x_t$ is defined by

$$x_t = \frac{\int_M w_{j,t}^M dj}{\int_M w_{j,t}^M dj + \int_H w_{j,t}^H dj},$$  \hspace{1cm} (45)

and follows a jump-diffusion process

$$dx_t = \mu_{x,t} dt + \sigma_{x,t} dB_t + \sigma_{y,x,t} dB_{t}^y + \sigma_{x,t}^{J} dJ_t.$$  \hspace{1cm} (46)

Coefficients in equation (46) depend on agents’ optimal policies and are specified in the appendix in Lemma 2. Here I just note that in equilibrium $\sigma_{x,t} \equiv 0$, so $x_t$ is driven only by two shock.

Markov equilibrium of the model is fully characterized once we know the functions $\xi^H(x,y)$, $\xi^M(x,y)$, $p(x,y)$, $\pi(x,y)$, $\pi^y(x,y)$, $\pi^J(x,y)$, $\mu_x(x,y)$, $\sigma_{y,x}(x,y)$ and $\sigma_{x}^{J}(x,y)$. Appendix D shows how these functions are characterized and solved for numerically.

5 Quantitative Results

Now I proceed to the numerical solution and properties of the economy. Table 4 summarizes the set of parameters along with their values used in the main calibration. For a subset of standard parameters I use the values commonly used in the literature. Time $t$ is measured in years. I set the parameter of relative risk aversion $\gamma$ to 10 and elasticity of intertemporal substitution $\psi^{-1}$ to 2, values frequently used in the asset pricing literature (see e.g. Bansal and Yaron, 2004). Using recursive preferences helps to achieve a stable risk-free rate. The EIS above one implies that the stock price responds positively to shocks to exogenous growth $y$. Time discount $\rho$ is set to 0.067 to keep the risk-free rate at the average level of 1%. Volatility of the aggregate productivity
Table 4: Calibration

<table>
<thead>
<tr>
<th>Preferences</th>
<th>( \gamma )</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>EIS</td>
<td>( 1/\psi )</td>
<td>2</td>
</tr>
<tr>
<td>Time discounting</td>
<td>( \rho )</td>
<td>0.067</td>
</tr>
</tbody>
</table>

**Innovation Technology**

| Innovation productivity, incumbent  | \( \chi^I \)  | 7.44 |
| Innovation productivity, entrant    | \( \chi^E \)  | 0.61 |
| Elasticity of innovation, incumbent| \( \alpha^I \) | 0.89 |
| Elasticity of innovation, entrant   | \( \alpha^E \) | 0.89 |
| Radical quality improvement        | \( \lambda \)  | 1.1  |
| Jump intensity                      | \( \bar{h} \)  | 0.4  |
| Innovation productivity             | \( s \)        | 0.5  |
| Share going to new managers         | \( \beta \)    | 0.5  |

**Shocks**

| Aggregate volatility                | \( \sigma \)  | 10%  |
| Mean reversion of exogenous growth  | \( \kappa_y \) | 0.225|
| Shock to exogenous growth volatility| \( \sigma_y^y \) | 0.0143|
| Non-diversifiable share             | \( \phi \)    | 0.4  |

**Incremental Growth: \( \iota = Ag^2 \)**

| Productivity (inverse)              | \( A \)       | 266  |

Shock \( \sigma \) is set to 10% to roughly match the standard deviation of aggregate dividend growth. Parameters of exogenous growth process \( \kappa_y \) and \( \sigma_y^y \) are set to match discrete-time counterparts reported in Bansal and Yaron (2004), Table 1.

Fraction of net worth that managers invest in their own firms \( \phi \) plays an important role in the model. When \( \phi \) equals zero managers are effectively identical to households. Positive values of \( \phi \) make them exposed to non-diversified risk which impacts optimal portfolio choice and the amount of innovation in the economy. In the baseline calibration I use \( \phi = 0.4 \). This value corresponds to the average share of net worth invested in private equity by households with active business ownership in the Survey of Consumer Finances (see Moskowitz and Vissing-Jørgensen, 2002). I do a comparative statics analysis with respect to the value of \( \phi \) to illustrate the effect of non-diversifiable risk on equilibrium. According to the data reported in Hall and Woodward (2010), Table 1, on average 50% of the value of successful start ups, backed by VC, remains in the hands of the founders. The rest goes to outside investors, so I set \( \beta = 0.5 \).

I set gradual innovation technology to have quadratic functional form \( \iota = Ag^2 \), which makes numerical algorithm tractable and provides sufficient flexibility to match the moments described below. Parameters \( \alpha^I \) and \( \alpha^E \) are set to 0.89. This value implies that
elasticities of radical innovation for both incumbents and entrants are within the range of estimates reported by Griliches (1990). Parameter $s$ is set to 0.5. It gives entrants and incumbents equal chances to succeed conditional on a preliminary success in radical innovation. I set parameter $\lambda$ to be equal to 1.1. Garcia-Macia et al. (2015) estimate average increase in quality of existing varieties due to drastic innovation to be around 7.5%. Acemoglu et al. (2013) estimate this parameter to be around 14.8%. The value I use lies near the middle of this range. Remaining technological parameters of innovation ($\chi^I, \chi^E, A, \bar{h}$) are set jointly to match a set of moments in the data. To keep the matter concrete, I consider the set of public firms as the real-world counterpart of incumbents, and venture capital industry as the counterpart of the innovation sector. I choose $\chi^I, \chi^E, A$ to match average incumbents’ R&D to incumbents’ operating profit ratio, average entrants’ R&D to incumbents’ operating profit ratio and average incumbents’ exit rate. In the baseline case innovation waves arrive on average once every two and a half years $\bar{h} = 0.4$. The frequency of wave arrival determines lumpiness of the innovation process which is one of the key assumptions that differentiate the present model from the previous literature. Lower values of $\bar{h}$ make investments in the innovation sector more risky. The value I use in the baseline calibration delivers a good match for the volatility of returns in the innovation sector and returns in venture capital observed in the data. In the analysis below I also do comparative statics with respect to this parameter to show its impact on solution properties.

**Properties of the Equilibrium** The upper panel of Figure 3 shows equilibrium amount of investment in radical innovation by entrants on the left and by incumbents on the right. Here I plot all quantities as functions of managers’ share $x$. State variable $y$ is set to be equal to 0 (solid), $-0.041$ (dash) and 0.041 (dash-dot) which correspond to 0.025, 0.5 and 0.975 quantiles of its stationary distribution. Investment rate in radical innovation by both incumbents and entrants increases both with $x$ and $y$. The bottom panel shows probabilities of successful innovation in a given product line conditional on arrival of a wave $\bar{\omega}^I$ and $\bar{\omega}^E$. Alternatively, $\bar{\omega}^I$ represents the share of firms that succeed

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15Griliches (1990) surveys the studies that estimate elasticity of R&D output, measured by patents, in response to changes in R&D.

16A substantial part of the economic activity is conducted by firms which are not publicly traded. At the same time venture capital constitutes only a part of the innovation sector. Given lack of data for broader definitions of the productive and innovation sectors I limit my calibration to these economically important definitions. The data on public firms comes from Compustat, data on VC flows is from VentureXpert. I limit the sample to 1993–2016, period when flows to VC have stabilized to exhibit stationary behavior.

17Exits are not directly observable in Compustat or CRSP. I proxy exits as a loss of 90% of market capitalization during one year.
Figure 3: **Investment in and probabilities of radical innovation.** Top: investment in radical innovation by entrants $\hat{z}^E$ and incumbents $\hat{z}^I$. Bottom: probabilities of successful radical innovation by an entrant $\overline{w}^E$ and incumbent $\overline{w}^I$. Solid line is for $y$ being equal to 0 or its median value, dash and dash-dot are for $y$ being equal to 0.025 and 0.975 quantiles of its stationary distribution.

in radical innovation and $\overline{w}^E$ represents the share of firms that get displaced. Increasing pattern of investment by the innovation sector $\hat{z}^E$ is driven by the different portfolio allocations chosen by managers and households. Incumbent firms increase investment in radical innovation with $x$ in response to more aggressive activity by entrants.

In the upper panel of Figure 4 I plot agents’ exposure to the jump risk as functions of managers’ share $x$ (left) and $y$ (right). Exposure to the jump risk is defined as a proportional change in wealth upon arrival of the innovation wave. Dash line denotes households’ exposure. For managers I plot two measures of exposure to the jump risk. Denoted by the solid line, $\tilde{\sigma}_{w,M}^{J,M}$ is exposure to jump of the liquid part of manager’s portfolio. Dash-dot line plots average across managers exposure to jump of the total net worth $\sigma_{w,M}^{J,M}$

$$\sigma_{w,M}^{J,M} = (1 - \phi) \tilde{\sigma}_{w,M}^{J,M} + \phi \sigma_{S}^{J}.$$

Here $\sigma_{S}^{J}$ is the expected instantaneous return on a single stock conditional on the arrival
Figure 4: **Portfolio loadings on jump risk and flow investment in the innovation sector.** Upper panel: managers’ liquid wealth loading on jump risk \( \tilde{\sigma}_{w}^{J,M} \), managers’ total wealth loading on jump risk \( \sigma_{w}^{J,M} \), households’ loading on jump risk \( \sigma_{w}^{J,H} \). Lower panel: investments in the innovation sector by managers and households normalized by personal wealth, \( \theta_{M} \) and \( \theta_{H} \) correspondingly. Left (right) column keeps state variable \( x (y) \) constant at its median value.

of a wave (which coincides with the aggregate stock market exposure to the jump risk) defined by

\[
\sigma_{S}^{J} = \left(1 - \bar{\omega} - \bar{\omega}^{E}\right) \times \sigma_{p}^{J} + \bar{\omega}^{J} \times \left(\lambda(1 + \sigma_{p}^{J}) - 1\right) - \bar{\omega}^{E}.
\]

The first term reflects the change in market value of firms that are not directly influenced by the innovation wave, the second corresponds to the firms that succeed in radical innovation and the third accounts for the firms that exit. Both managers’ liquid and total wealth are significantly more exposed to jump risk relative to households’ wealth. Each manager faces a considerable probability of her business being disrupted by an entrant conditional on arrival of the innovation wave. To hedge this risk managers expose their liquid wealth to more of the jump risk.

A different way to look at agents’ portfolio choice is to replicate risk exposures using
the set of tradable securities: the stock market, the innovation sector, security-$y$ and the risk-free asset. Among them, the stock market is the only security that provides exposure to the aggregate productivity shock $dB_t$. So managers’ portfolio allocation in the stock market is simply given by $(1 - \phi)\tilde{\sigma}_w^M / \sigma_S$, and households’ is given by $\sigma_w^H / \sigma_S$. Investment flows, normalized by net worth, into the innovation sector for households and managers are defined respectively by

$$
\theta^H = \bar{h}(1 - \pi^J)\left(\sigma^J_w^H - \sigma^J_S \times \frac{\sigma^H_w}{\sigma_S}\right),
$$  \hspace{1cm} (49a)

$$
\theta^M = (1 - \phi)\bar{h}(1 - \pi^J)\left(\tilde{\sigma}^J_w^M - \tilde{\sigma}^J_S \times \frac{\tilde{\sigma}_w^M}{\sigma_S}\right). \hspace{1cm} (49b)
$$

The bottom panel of Figure 4 plots investment flows into the innovation sector by managers and households. Each manager invests significantly more in the innovation sector relative to her net worth compared to households. Interestingly each manager’s flow investment in the innovation sector $\theta^M$ is increasing with the state variables $x$ and $y$ under this calibration. This implies that increasing in $x$ pattern of aggregate investment in the innovation sector observed on Figure 3 is driven by two forces. First, managers’ allocation to the innovation sector is higher than households’ per unit of wealth. Second, each manager faces higher risk of disruption and invests even higher share of own wealth in the innovation sector when $x$ is higher. The second channel illustrates the feedback loop between the amount of innovation activity and agents’ portfolio choice.

As it follows from Figure 4 households’ flow investment in the innovation sector gradually declines with $x$. This effect is driven by the decline in price of jump risk plotted on the left panel of Figure 5. The price of jump risk equilibrates demand and supply.

Figure 5: **Price of jump risk and expected distributions to paid-in capital ratio.** Solid lines are for $y$ being equal to 0 or its median value, dash and dash-dot are for $y$ being equal to 0.025 and 0.975 quantiles of its stationary distribution.
Figure 6: **Dynamics of $x$**. State variable $x$ follows a jump-diffusion process $dx_t = \mu_x \, dt + \sigma^y_x \, dB^y_t + \sigma^J_x \, dJ_t$. Right panel plots kernel density of $x$ obtained using a 10000 years simulation of the model. Solid lines are for $y$ being equal to 0 or its median value, dash and dash-dot are for $y$ being equal to 0.025 and 0.975 quantiles of its stationary distribution.

Higher $x$ leads to more demand for positive exposure to jump risk stemming from the managers willing to hedge against disruption. At the same time net supply of the jump risk is more likely to decline with $x$ due to displacing effect of the innovation sector. The right hand side of Figure 5 plots the expected distributions to paid-in capital multiple (DPI) for the innovation sector in the model. DPI is one of the most popular and important performance measures in the Venture Capital industry. Realized DPI represents the ratio of total distributions that investors in VC receive to the capital invested. In my model expected DPI is related to the price of jump risk via a simple equation

$$ \text{Expected DPI} = \frac{1}{1 - \pi^J}. \quad (50) $$

Lower price of risk results in lower expected return on investments in the innovation sector. When the price of risk equals zero expected DPI equals one. In other words on average for every dollar invested agents receive one dollar back. In the majority of expanding variety or Schumpeterian growth models, that do not feature aggregate uncertainty, the DPI ratio is deterministic and always equals one. In the current setup expected DPI is a decreasing function of $x$. Innovation sector provides a hedge against disruption, hence expected returns fall when the risk of disruption is higher.

**Dynamics of the Economy** Now I explore the dynamic properties of the model. The state of the economy is described by one exogenous state variable, growth $y$, and one endogenous state variable, share of wealth owned by the managers $x$. Managers’ share
follows the process
\[ dx_t = \mu_{x,t}dt + \sigma^y_{x,t}dB^y_t + \sigma^J_{x,t}dJ_t. \]  
(51)

The drift component \( \mu_{x,t}dt \) determines expected change of \( x \) in absence of innovation waves. The second term represents response of \( x \) to shocks in growth \( y_t \). The third term, \( \sigma^J_{x,t}dJ_t \), governs change in \( x \) upon arrival of the innovation wave. Note that in the above equation I omitted the term \( \sigma_{x,t}dB_t \) due to the fact that \( \sigma_{x,t} \equiv 0 \). The first three panels of Figure 6 plot each component of the right hand side of equation (51). The rightmost panel plots the probability density function of \( x \) obtained by simulating the economy for 10000 years. Quantitatively the drift \( \mu_{x,t}dt \) and jump \( \sigma^J_{x,t}dJ_t \) components are the key drivers of \( x_t \).

The magnitude of drift \( \mu_{x,t} \) is affected, first, by the difference in consumption rates by managers and households and, second, by the difference in investment rates in the innovation sector. The drift is consistently negative mostly due to managers’ higher investment in innovation \( \theta^M > \theta^H \) that we saw on Figure 4 and a higher rate of consumption.

The sign of \( \sigma^y_{x,t} \) is consistently positive. It is driven by the fact that managers choose higher loading on \( dB^y_t \) shock in their investment portfolio relative to households. As we saw on Figure 3 higher \( y_t \) corresponds to a higher rate of creative destruction — a state relatively more harmful for managers compared to households. Hence managers hedge against the increase in \( y_t \). Quantitatively this effect is small. One standard deviation shock to \( dB^y_t \) leads to increase in \( x_t \) by no more than 15 basis points.

The magnitude of jump \( \sigma^J_{x,t} \) is influenced, first, by exit of displaced managers and entry of the new ones, and, second, by the difference in wealth exposures to the innovation risk between households and managers. As it follows from Figure 4 jump in \( x \) is positive when \( x \) is below a certain threshold for a given value of \( y \). In this region managers as a group gain wealth relative to households due to taking larger exposure to jump risk (Figure 4). The effect of entry and exit on \( x \) is positive for smaller values of \( x \) and negative for larger values of \( x \). When \( x \) is sufficiently high, \( \sigma^J_{x,t} \) is negative and managers as a group own so much wealth that arrival of a wave reduces their share due to exit of wealthy managers and entry of more poor ones. As it follows from the rightmost panel, in equilibrium the economy spends most of the time in the region \( x \in (0.1, 0.6) \), where \( \sigma^J_{x,t} > 0 \).

Figure 7 illustrates a simulation of the economy for 200 years. Shaded bars correspond to years with one or more wave arrivals. The first panel shows dynamics of the state variable \( x \). Consistent with the discussion above, managers’ share goes up in periods,
when innovation waves arrive, and tends to go downwards in their absence. Panels two and three plot dynamics of investment in radical innovation by entrants and incumbents correspondingly. These two series exhibit highly volatile behavior at the frequencies of up to ten years. This volatility is driven by the exogenous shock to growth $y$. Positive shock to $y$ leads to the increase in the stock price $p_t$. Higher stock prices make investment in both gradual and radical innovation more attractive. This effect leads to an increase in the risk of disruption and decrease in the price of jump risk $\pi^J$ and expected return on innovation, which I plot in the last panel. Drop in the price of jump risk amplifies the effect of the positive shock to $y$ and leads to even more investment in radical innovation.

At lower frequencies, of the order of decades, there is a clear comovement between the amount of radical innovation and managers’ share $x$. This correlation gives rise to the endogenous long-run innovation cycles. A typical cycle starts with a sequence of
Figure 8: **Aggregate stock market exposure to shocks.** Return on the aggregate stock market is given by the jump diffusion $dR_t = \mu R_t dt + \sigma_R^y dB^y_t + \sigma_R^J dJ_t$, where $\sigma_R$ is constant in equilibrium and is equal to the aggregate productivity shock volatility $\sigma$. Solid lines are for $y$ being equal to 0 or its median value, dash and dash-dot are for $y$ being equal to 0.025 and 0.975 quantiles of its stationary distribution.

innovation waves that bring managers’ share $x$ up. The price of jump risk drops leading to more investment in radical innovation.

Table 5: **Innovation and the stock market.** The table presents results of linear regressions of risk measures $Y_t$ on innovation activity $IA_t$: $Y_t = \text{Const} + \beta IA_t + \epsilon_t$. Regressions are based on a 10000 years simulation of the model. Simulation is at the monthly frequency and all variables are aggregated to the annual level. Innovation activity is defined as $\iota + \hat{z}I + \hat{z}E$. CS StDev is the cross-sectional standard deviation of annual firm-level stock returns; Exit Rate is the share of firms that go out of business in a given year; Market Volatility is the realized volatility on aggregate stock market in year $t$.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>CS StDev</th>
<th>Exit Rate</th>
<th>Market Volatility</th>
</tr>
</thead>
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<tr>
<td>$IA_t$</td>
<td>0.0111</td>
<td>0.0035</td>
<td>0.0004</td>
</tr>
<tr>
<td>Mean($Y_t$)</td>
<td>0.0677</td>
<td>0.0119</td>
<td>0.1055</td>
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**Innovation and the Stock Market**  Time-varying innovation activity in the model economy generates time-varying volatility of both aggregate and individual stock returns. Volatility of the aggregate stock market varies due to time-varying exposure to aggregate shocks. At the individual firm level innovation waves create extra risk due to their unpredictable heterogeneous effect across firms. In Table 5 I present linear regressions of measures of risk on aggregate innovation activity. Regressions are based on a single 10000 years simulation of the economy. I define innovation activity as the sum of all R&D expenditures in the economy $IA_t = \iota_t + \hat{z}I + \hat{z}E$ and normalize it to have zero mean.
and unit standard deviation in the sample. Along with the slope coefficients I report mean values of dependent variables.

In the first two columns I report results for two measures of idiosyncratic risk, namely cross-sectional standard deviation of annual returns and share of firms exiting in a given year. One standard deviation increase in $IA_t$ is associated with a large increase in idiosyncratic risk. Cross-sectional standard deviation goes up by 1.1 percentage point or roughly 16% relative to its average value. Exit rate goes up by 35 basis points or 29% of its mean value. The mechanism behind this result is clearly seen on Figure 3: both the exit rate $HE$ and radical innovation spendings $z^E$ and $z^I$ increase in each of the state variables $x$ and $y$. This gives rise to the positive correlation between innovation and idiosyncratic risk.

The last column in Table 5 shows regression of the aggregate stock market volatility on $IA_t$. Aggregate volatility is measured as realized volatility of the aggregate market portfolio in a given year. The slope coefficient is very small in absolute value (and statistically insignificant in this simulation), so quantitatively there is virtually no relationship between the two variables. In addition, mean realized volatility is only slightly higher than the volatility of aggregate productivity shock $\sigma$. To understand this result better, recall that aggregate stock market return is given by $dR_t = \mu_{R,t}dt + \sigma_{R,t}dB_t + \sigma_y^u dB_t^y + \sigma_I^I dJ_t$. In equilibrium it turns out that $\sigma_{R,t}$ does not vary in time and is equal to $\sigma$. Exposure to the second Brownian shock $\sigma_y^u$ is plotted on the left panel of Figure 8. Its magnitude relative to $\sigma$ (which equals 0.1 in my calibration) and time-variation are small. The last contributor to the aggregate stock market volatility is $\sigma_I^I dJ_t$. I plot $\sigma_I^I$ on the right panel of Figure 8. The graph implies that the stock market almost always appreciates with an arrival of the innovation wave. Quantitatively it adds little to the aggregate stock market volatility though: $\overline{h} \times \sigma_I^I$ rarely exceeds 1.5 percentage points. Hence consistent with empirical observations, times of high innovation activity in a substantial manner redistribute market value between the firms while keeping aggregate stock market returns only mildly affected.

**Strategic Complementarity** The sharp reaction of the investment in the innovation sector to changes in state variables to large extent is driven by the strategic complementarity of managers’ decisions to invest in the innovation sector. To abstract from general equilibrium effects I demonstrate this feature by solving the manager’s problem (42) in
Figure 9: Individual manager’s best response to aggregate investment rate in the innovation sector by managers $\Theta^M$.

In a partial equilibrium setting. In particular, I fix the quantities $x_t$, $y_t$, $p_t$, $\sigma^J_{p,t}$, $\sigma^J_{S,t}$, $\pi^I_t$, $\xi^M_t$, $\xi^H_t$, $\sigma^J_{\xi,t}$, $\sigma^J_{\xi,t}$ and solve the individual manager’s problem (42) for varying values of aggregated investment in the innovation sector by managers as a group normalized by their wealth. I denote this quantity by $\Theta^M_t$. Investment in the innovation sector by managers feeds into the problem (42) through probabilities of successful radical innovation by the incumbent $\overline{\pi}^I(\hat{z}^I_t, \hat{z}^E_t)$ or an entrant $\overline{\pi}^E(\hat{z}^I_t, \hat{z}^E_t)$, where

$$
\hat{z}^I_t = \frac{x_t p_t Q_t \Theta^M_t + \int_j \theta^H_{j,t}(\hat{z}^I_t, \hat{z}^E_t) dj}{Q_t}.
$$

Note that households’ investment rates in the innovation sector $\theta^H_{j,t}$ do not vary with $\Theta^M_t$ since they depend on $\pi^I_t$ and $\sigma^J_{\xi,t}$ only (see formula 41). Figure 9 plots best responses of an individual manager to the value of $\Theta^M_t$. Both $\theta^M_{i,t}$ and $\hat{z}^I_{i,t}$ increase in $\Theta^M$. Higher $\Theta^M$ implies a higher probability of disruption. Hence the manager allocates more resources both within the firm and through portfolio choice to hedge this risk. Positive response of $\theta^M_{i,t}$ to $\Theta^M_t$ represents a strategic complementarity. This relationship gives rise to the high volatility of the innovation activity in the economy. For example, increase in expected growth $y_t$ leads to higher stock prices. Higher stock prices attract more investment in innovation. As a result, managers increase portfolio allocations to the innovation sector to both take advantage of the higher price and in attempt to hedge the higher risk of disruption.

Innovation by entrants unambiguously reduces managers’ welfare. Hence, apart from generating high volatility, this complementarity exacerbates the business stealing effect.
of disruptive innovation. Managers hedging behavior as a group accelerates the process of creative destruction and reduces their welfare.

The Effect of Lumpiness of the Innovation Process  Unlike the majority of economic growth models with expanding varieties or quality ladders, this model features lumpy arrival of innovations that make investments in radical innovation risky at the aggregate. This feature allows me to study expected returns on investment in the innovation sector and has significant impact on the intensity and time-variation of the innovation process. In this section I do comparative statics with respect to the frequency of innovation wave arrivals $\bar{h}$. To isolate the effect of wave arrival intensity from the productivity of innovation technology I modify the investment technology (10c–10d) according to:

$$
\omega^E_{i,t} = \frac{h_0}{\bar{h}} \times \omega^E_{i,t} \left(1 - \omega^I_{i,t}s\right),
$$

$$
\omega^I_{i,t} = \frac{h_0}{\bar{h}} \times \omega^I_{i,t} \left(1 - \omega^E_{i,t}(1 - s)\right),
$$

where $\omega^I_{i,t}$ and $\omega^E_{i,t}$ are still defined by (10a–10b). Here $h_0$ is a fixed constant that I set to be equal to 0.4 (on average waves arrive every 2.5 years), the value of $\bar{h}$ in the baseline calibration. This specification possesses a couple of properties which are important for the comparative statics exercise. First, when $\bar{h} = h_0$ this technology coincides with the original one, specified by equations (10a–10d). Second, keeping $h_0$ constant and increasing $\bar{h}$ leads to an increase in the frequency of aggregate wave arrivals. However, probability of successful innovation in an instant of time $dt$ by either incumbent or an entrant remains unchanged, provided investment rates $\hat{\imath}^I_{i,t}$ and $\hat{\imath}^E_{i,t}$ are kept constant. This means that higher $\bar{h}$ increases intensity of aggregate innovation arrivals and proportionally reduces share of firms that succeed in radical innovation $\omega^E_{i,t}$ or are replaced by entrants $\omega^I_{i,t}$. In the limiting case $\bar{h} \to +\infty$ this technology effectively converges to the one with idiosyncratic arrivals of radical innovations at the firm level, a common assumption in the models of economic growth.\(^\text{20}\)

Figure 10 plots equilibrium quantities in (i) the baseline economy with $\bar{h} = 0.4$ (solid line), (ii) economy with waves arriving on average once a year (dashed line), (iii) waves arriving on average twice a year (dash-dot line) and (iv) infinite arrival intensity (dotted line). The upper panel plots the price of jump risk $\pi^I$. The profile of $\pi^I$ has the steepest (negative) slope for $\bar{h} = 0.4$ both as a function of managers’ share $x$ and as a function

\(^{20}\)Chapters 13 and 14 in Acemoglu (2008) provide a good overview.
Figure 10: **Comparative statics with respect to jump intensity** \( \bar{h} \). Upper row: price of jump risk \( \pi^J \); lower row: investment in the radical innovation by entrants \( \hat{z}_E \). Left column plots variables as functions of managers’ share \( x \) keeping state variable \( y \) constant and equal to 0, its median value. Right column plots variables as functions of exogenous component in expected growth \( y \) keeping \( x \) constant and equal to its median value in the baseline calibration. Solid line is for \( \bar{h} = 0.4 \), dashed for \( \bar{h} = 1 \), dash-dot for \( \bar{h} = 2 \), dotted for \( \bar{h} = +\infty \).

of exogenous component of expected growth \( y \). The slope gradually declines with \( \bar{h} \). With lower rate of wave arrival the innovation sector provides a good hedge for managers in case of disruption to their business. When waves are more frequent share of firms disrupted in each wave goes down. This makes aggregate jumps less correlated with displacement of any given manager. Hence exposure to the jump risk becomes a worse hedge, and managers’ demand for jump exposure goes down. The bottom row plots investment in R&D by entrants \( \hat{z}_E \). Behavior of \( \hat{z}_E \) effectively mirrors the profiles of \( \pi^J \): the slope flattens out for higher values of \( \bar{h} \). In the \( \bar{h} \to +\infty \) case, when the price of jump risk equals zero, the slope of \( \hat{z}_E \) as a function of \( x \) even turns negative. This effect is driven by the positive dependence of the risk-free rate on \( x \).\(^{21}\)

\(^{21}\)When EIS is higher than one, presence of idiosyncratic risk makes managers less patient relative to households. Hence risk-free rate is an increasing function of \( x \).
Figure 11: **Comparative statics with respect to non-diversifiable share** \( \phi \). Upper row: price of jump risk \( \pi^J \); lower row: investment in the radical innovation by entrants \( \hat{z}^E \). Left column plots variables as functions of managers’ share \( x \) keeping state variable \( y \) constant and equal to 0, its median value. Right column plots variables as functions of exogenous component in expected growth \( y \) keeping \( x \) constant and equal to its median value in the baseline calibration. Solid line is for \( \phi = 0.4 \), dashed for \( \phi = 0.35 \), dash-dot for \( \phi = 0.3 \), dotted for \( \phi = 0 \).

**The Effect of Non-diversifiable Risk** On Figure 11 I compare solutions of the model for different values of non-diversifiable share of managers’ wealth \( \phi \). I plot the price of jump risk in the upper panel and investment in R&D by entrants in the lower one. In both panels I plot variables as functions of each state variable \( x \) and \( y \) while keeping the remaining state variable at its median value in the baseline calibration with \( \phi = 0.4 \). The price of jump risk \( \pi^J \) is a decreasing function of managers’ share \( x \). The absolute value of its slope, however, sharply goes down for lower \( \phi \). This effect is reflected in a weaker response of \( \hat{z}^E \) to changes in \( x \) observed in the lower panel. For lower values of \( \phi \) managers are less worried about the risk of disruption. Hence as a group, they demand lower amount of insurance against displacement. This effect makes their portfolio look more like the portfolio chosen by households and dependence of \( \pi^J \) and \( \hat{z}^E \) on \( x \) becomes milder.
As can be seen on the upper left graph of Figure 11, in the baseline calibration $\pi^J$ is a decreasing function of managers’ share. Higher expected growth $y$ implies higher prices on stocks. High prices stimulate entry of new firms and more demand for insurance against disruption as a result. For lower values of $\phi$ the dependence of $\pi^J$ on $y$ turns from negative to positive. The easiest way to understand this result is to consider $\phi = 0$ case, when managers are effectively identical to households. Arrival of an innovation wave is good news for all agents in the economy, since the wave brings more value than destroys. Higher $y$ implies higher stock prices in the $\text{EIS} > 1$ case and arrival of a wave brings proportionally more value, so the price of jump risk becomes even higher.

In summary, comparative statics with respect to $h$ and $\phi$ have shown that both lumpy arrival of innovations and lack of diversification are responsible for the economy’s Sharpe response to changes in state variables.

6 Puzzles in Venture Capital

In this section I contrast predictions of the model with the data on the venture capital industry, a close empirical counterpart of the innovation sector in the model. The data on fund flows and financial performance of venture capital funds allows me to investigate the model’s ability to explain jointly the asset pricing properties and the volumes of innovation activity.

Empirical literature has documented a set of stylized facts related to the venture capital activity.$^{22}$ Some of these facts remain to be puzzling for classical theories. I will look at these empirical observations through the lense of my model to both test the model’s implications and give potential explanations to the pervasive patterns.

**Measuring Financial Performance in VC** Longer investment horizons and unobservability of portfolio values in intermediate periods complicate measuring performance of investments in venture capital. Hence approaches to measuring performance in VC differ from the methods used for publicly traded securities. Below I give a brief overview and definitions of the measures I use in my analysis.

In VC, the three most intuitive and popular among researchers and practitioners measures of investment performance are (i) internal rate of return or IRR, (ii) public market equivalent or PME, and (iii) investment multiples, such as distributions to paid-in

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$^{22}$Empirical studies have uncovered stylized facts and patterns both at the aggregate industry and individual fund level. Since my model treats the innovation sector as a centralized intermediary I will focus on the aggregate industry level evidence only.
capital (DPI) and the total value to paid-in capital (TVPI). I focus on TVPI and the PME measure of Kaplan and Schoar (2005). These two measures can be defined in my model with minimal additional assumptions which I describe in detail below.\(^\text{23}\)

For a VC fund raised in a given year TVPI is defined by

\[
TVPI = \frac{\text{Distributions} + \text{Residual value}}{\text{Paid-in capital}}.
\]  

(54)

Paid-in capital is the total amount of funds that have been called and invested by general partners during the fund’s life. Distributions is the total amount of funds that has been paid back to investors. Residual value is the value of assets still in the fund’s portfolio. By the end of fund’s life residual value equals zero and all equity is paid back to limited partners. A typical lifetime of a VC fund is ten years. Capital is usually called during the first five years, followed by five years of divestment. To define TVPI on VC sector in the model I assume, first, that capital is invested uniformly in time during the first five years of funds’ life. Second, conditional on successful innovation, new firms are held in the portfolio for five more years. For a cohort of funds of vintage \(t\) the final pooled TVPI multiple thus equals

\[
TVPI_t = \frac{\text{Distributions}}{\text{Paid-in capital}} = \frac{\int_t^{t+5} P_u \times \frac{R_{u+5}}{R_u} \times dJ_u}{\int_t^{t+5} 1 \times du}.
\]  

(55)

In the formula above, \(P_u\) is the payoff on investment in the innovation sector at a rate of one dollar a year, at time \(u\), conditional on wave arrival

\[
P_u = \frac{1}{(1 - \pi_u)^h}.
\]  

(56)

The multiple \(R_{u+5}/R_u\) is the five-year return on the stock market. It accounts for the five year holding period of the portfolio companies. Note that TVPI multiple ignores an important notion of the time value of money. PME measure suggests a way to address this issue.

For a given fund Kaplan and Schoar (2005) define PME as the ratio of the sum of discounted distributions to the sum of discounted capital calls. The discount rate between dates \(u\) and \(s\) is the gross realized return on a benchmark portfolio between dates \(u\) and \(s\). PME for an investment in the benchmark at any horizon always equals

\(\text{For fully divested funds DPI coincides with TVPI. IRR measure is not defined for some realizations of the cashflow stream. One such scenario is a cashflow with positive capital calls and zero realized distributions, a scenario that occasionally happens in the model.}\)
Table 6: **Venture capital flows and performance summary.** The table reports summary statistics of fund flows into VC, TVPI multiple and PME. Mean is average value, SD is standard deviation, IQR is interquartile range. In the data flows are defined as aggregate VC funds commitments in the US in a given year, source: VentureXpert by Thomson. Earnings are the total annual earnings of public firms, source: Compustat. TVPI is total value to paid-in multiple of the funds raised in a given year, PME is public market equivalent of Kaplan and Schoar (2005). Both performance measures are calculated by Brown et al. (2015) using Burgiss data. Standard errors for mean and standard deviation are Newey-West and standard errors for IQR are based on block bootstrap. The data is annual, 1993–2010. The model counterparts are obtained in a 18000 year simulation of the model.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean(Flow/Earnings)</strong></td>
<td>0.072 (0.020)</td>
<td>0.060</td>
</tr>
<tr>
<td><strong>SD(Flow/Earnings)</strong></td>
<td>0.057 (0.015)</td>
<td>0.017</td>
</tr>
<tr>
<td><strong>IQR(Flow/Earnings)</strong></td>
<td>0.039 (0.035)</td>
<td>0.019</td>
</tr>
<tr>
<td><strong>Mean(TVPI)</strong></td>
<td>2.623 (0.808)</td>
<td>2.301</td>
</tr>
<tr>
<td><strong>SD(TVPI)</strong></td>
<td>2.105 (0.498)</td>
<td>1.731</td>
</tr>
<tr>
<td><strong>IQR(TVPI)</strong></td>
<td>1.940 (1.564)</td>
<td>2.171</td>
</tr>
<tr>
<td><strong>Mean(PME)</strong></td>
<td>1.641 (0.414)</td>
<td>1.254</td>
</tr>
<tr>
<td><strong>SD(PME)</strong></td>
<td>1.069 (0.232)</td>
<td>0.889</td>
</tr>
<tr>
<td><strong>IQR(PME)</strong></td>
<td>1.330 (0.756)</td>
<td>1.220</td>
</tr>
</tbody>
</table>

one. In practice most popular benchmarks are portfolios of broad equity indices such as S&P500 or Russell indices. A natural benchmark index in my model is the aggregate portfolio of incumbent firms. Hence to define PME in the model I discount cash flows in the nominator and denominator of formula (55) to date $t$ by the stock market return

$$
\text{PME}_t = \frac{\text{Discounted distributions}}{\text{Discounted paid-in capital}} = \frac{\int_t^{t+5} P_u \frac{R_{u+5}}{R_u} dJ_u}{\int_t^{t+5} 1 \times \frac{R_t}{R_u} du} = \frac{\int_t^{t+5} P_u \frac{R_t}{R_u} dJ_u}{\int_t^{t+5} \frac{R_t}{R_u} du}. 
$$

(57)

PME has an intuitive interpretation: $(\text{PME}_t - 1) \times 100$ corresponds to the percentage point outperformance of the VC funds of vintage $t$ relative to the identically timed investment in the stock market.

**Fund Flows and Performance: Level and Volatility** In the first three rows of Table 6 I report summary statistics of annual flows into venture capital in the US normalized by aggregate earnings of public corporations. The data on flows comes from VentureXpert by Thomson and data on earnings is from Compustat. First, the table shows that annual flows into VC are sizable: in 1993–2010 on average they corresponded
to about 0.07 of corporate earnings. Second, fund flows into venture capital are highly volatile. This feature of venture capital has been widely documented in the empirical literature. I report two measures of volatility: the standard deviation and the interquartile range to make the picture more complete given small sample size. Large ratio of the standard deviation to IQR in the data indicates a significant impact of extremely high fund flows observed in 1999–2001.

The third column in Table 6 reports the model counterparts of the summary statistics. These values are obtained in a 18000 years simulation of the model. The simulation is at the monthly frequency. In the model I define flows by the instantaneous value of $\hat{z}_t^E$ and earnings by the instantaneous value $(a - \nu_t - \hat{z}_t^I)$. By virtue of the choice of parameter $\chi^E$ the model matches well the average amount of flows into the innovation sector. The model seems to understate volatility of fund inflows which is not targeted directly in the calibration. The standard deviation of the fund flows is 0.017 in the model compared to 0.057 in the data. The IQR is 0.019 compared to 0.039 observed in the data.

First two columns in the middle and bottom panels of Table 6 report summary statistics of returns on investment in VC. TVPI and PME are the capital-weighted performance measures for North Ameralic venture funds calculated and reported by Brown et al. (2015). Mean value of TVPI in the sample equals 2.6 with the standard deviation of 2.1 and the IQR of 1.9. These numbers show that even at the aggregate industry level investments in VC are extremely risky. Recall that these numbers correspond to an investment at the ten year horizon. The mean value of PME equals 1.64. This indicates that on average a VC fund outperformed the stock market by 64 percentage points throughout its lifetime. Average value of PME is lower than TVPI since it makes an adjustment for the time value of money and a crude adjustment for the market risk.

The third column in Table 6 reports the model implied counterparts of TVPI and PME. The model suggests slightly lower average returns on VC as measured by PME and TVPI, although the measures lie well within the range of one standard error of empirical estimates. Remarkably, the model matches well the volatility of VC returns.

---

24 Data on VC flows and performance in my sample is available starting from 1984. Due to rapid growth of the industry and data collection efforts VC flows series exhibits highly non-stationary behavior in 80s and early 90s.


26 For a normally distributed random variable this ratio is 0.74.

27 Note that instantaneous value of $\hat{z}_t^E$ does not exactly represent fund commitments, which in practice are drawn down and invested over the course of several years. However, in my analysis I find this measure to be a better proxy for commitments compared to, for example, realized investments in the innovation sector over five years. Similar to fund commitments, the former measure is not forward looking (while the latter is) which is very important in forecasting regressions I conduct below.
Both standard deviation and IQR are very close to empirical estimates. This result implies that even a stylized way of modelling the aggregate risk of innovation, via pure jumps, delivers rich and realistic properties of returns.

Table 7: Fund flows and subsequent performance. The table reports slope coefficients in linear regressions of VC performance measures $Y_t$ on fund flows: $Y_t = \text{Const} + \beta \log(\text{Flows}/\text{Earnings})_t + \gamma t + \epsilon_t$. Standard errors are Newey-West. The data is annual, 1993–2010. The model counterparts are obtained in a 18000 year simulation of the model.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>SE</td>
</tr>
<tr>
<td>TVPI</td>
<td>-1.950</td>
<td>(0.421)</td>
</tr>
<tr>
<td>PME</td>
<td>-0.977</td>
<td>(0.154)</td>
</tr>
</tbody>
</table>

Fund Flows and Performance: Predictability. A range of empirical studies find strong relationship between capital commitments and financial returns in the VC. Gompers and Lerner (2000), Kaplan and Schoar (2005), Harris et al. (2014), Robinson and Sensoy (2016), among others, find that aggregate volume of VC commitments in a given year negatively correlate with performance of the VC funds raised in that year. Gompers and Lerner (1998), Kaplan and Schoar (2005), Harris et al. (2014) show that high aggregate returns in VC are followed by more capital inflows.

In the first two columns of Table 7 I report slope coefficients in OLS regressions of performance measures on the log of commitments to corporate earnings ratio. I include a linear time trend in these regressions to focus on cyclical fluctuations of investment activity. Consistent with previous studies I find a strong negative relationship between vintage fund flows and performance of funds of that vintage. A ten percent increase in fund flows corresponds to an average decrease in PME by 0.1, a 15% percent drop in excess return of VC over the stock market.\(^{28}\)

I run the same regressions in the simulated data (without a time trend) and report the results in the last column of Table 7. The model reproduces the data very well. PME and TVPI are forecasted with the correct sign and magnitudes lie very close to the empirical estimates. In the model, the price of innovation risk $\pi^J$ is the key determinant of expected returns on innovation. Times of high innovation activity are associated with a higher rate of disruption. In these circumstances managers are willing to pay more for a financial hedge against it and drive the risk premium on the innovation sector down. This effect directly translates into a decrease in expected returns on innovation and lower

\(^{28}\)Average PME of 1.641 implies an excess return of 64.1 percentage points, $0.0977/0.641 \approx 15\%$.  

45
average TVPI and PME. The described mechanism is consistent with Gompers and Lerner (2000) who find that “inflows of capital into venture funds increase the valuation of these funds new investments... Changes in valuations do not appear related to the ultimate success of these firms. The findings are consistent with competition for a limited number of attractive investments being responsible for rising prices.”

Table 8: VC performance and subsequent flows. The table reports slope coefficients in linear regressions of VC fund flows on lagged performance measured by PME: 
\[
\log(\text{Flows/Earnings})_t = \text{Const} + \beta \text{PME}_{t-k} + \gamma t + \epsilon_t \quad \text{for } k \text{ up to ten years.}
\] 
Standard errors are Newey-West. The data is annual, 1984–2010. The model counterparts are obtained in a 18000 year simulation of the model.

| Lag = 1  | 0.024 | (0.095) |
| Lag = 2  | 0.216 | (0.101) |
| Lag = 3  | 0.413 | (0.100) |
| Lag = 4  | 0.582 | (0.065) |
| Lag = 5  | 0.584 | (0.084) |
| Lag = 6  | 0.471 | (0.159) |
| Lag = 7  | 0.310 | (0.169) |
| Lag = 8  | 0.166 | (0.121) |
| Lag = 9  | 0.097 | (0.110) |
| Lag = 10 | 0.090 | (0.094) |

Table 8 reports results of the regressions of fund flows on lagged aggregated VC performance. I report results for PME only to save space, but the conclusions are robust to using TVPI or other measures of financial performance. I extend the sample backward up to 1984 to have enough data points to conduct a meaningful analysis for the lags of up to ten years. Estimates in the first column exhibit a clear hump-shaped dependence between performance of funds of past vintages and capital commitments to VC. Quantitatively the relationship is strong: a ten percentage point increase in PME of vintage \( t - 5 \) corresponds to the increase in inflows by about six per cent. I repeat the analysis using the data simulated from the model and report the results in the last column of Table 8. The model generates a similar hump-shaped pattern.

In the model past performance is related to the fund flows largely due to the time variation in managers’ wealth share \( x \). High returns on the innovation sector are driven by a sequence of innovation wave arrivals. Each arrival leads to an upward jump in the state variable \( x \). As we saw in the previous sections demand for exposure to jumps is higher for managers. Hence flows into the innovation sector increase with \( x \). This effect
generates a positive response of $\hat{z}^E$ observed in the last column of Table 8. Quantitatively this effect appears to be not strong enough to match the data. Alternative theories, based on additional assumptions (e.g. learning, slowly-moving capital or sentiment) have a potential to better match the pattern quantitatively.

Overall the model shows a good ability to replicate the empirical stylized facts qualitatively and in most cases quantitatively. Given the stylized structure of the risk of innovation the model delivers rich and realistic predictions on the distribution of aggregate returns in venture capital.

7 Evidence from the Cross-Section of Stocks

In this section I test asset pricing predictions of the model using the panel of publicly traded stocks. Using CRSP data I form portfolios that are likely to be heterogeneously exposed to the technological progress and test two predictions of the model. First, the model predicts a positive comovement between unexpected changes of idiosyncratic risk (driven by technological innovation) and returns on stocks that are likely to benefit from innovation. Second, I test if returns on these portfolios are predictable by the level of innovation activity in the economy and if predictability varies across the portfolios. The model implies that returns on stocks, that are more likely to appreciate with the arrival of a technological breakthrough, are lower in times of high innovation activity.

Every month I sort firms into five portfolios based on the number of years the firms have been publicly traded. Portfolio 1 includes stocks of the youngest firms and portfolio 5 — stocks of the oldest. Younger firms have higher valuations, higher R&D-to-revenue ratios, and often have high growth prospects. Older firms generally have more established business strategy with limited growth potential and higher exposure to the negative effects of disruption. To test the first hypothesis, outlined in the previous paragraph, I run a regression of monthly value-weighted excess returns on these portfolios against the shock to idiosyncratic risk. I measure idiosyncratic risk by the cross-sectional standard deviation of returns. The shock is defined as the difference between the cross-sectional standard deviation and its one-month lagged 12-month average value. For easier interpretation, I standardize the shock to have unit variance. I control for the contemporaneous market excess return to ensure the results are not driven by heterogeneity in market beta and the comovement between idiosyncratic risk with the aggregate market.

29 The actual firm age is difficult to measure. Years since IPO is a proxy for firm age. IPO date is defined either by the date reported in Compustat or the date of first observation in CRSP.
Table 9: Comovement of returns and idiosyncratic risk. The table presents results of linear regressions of value-weighted excess returns on shocks to idiosyncratic volatility controlling for market return. Portfolios are formed monthly based on the number of years since a firm has been publicly listed. Shocks to idiosyncratic risk are defined as the difference between the cross-sectional standard deviation of returns and one-month lagged 12-month moving average value. Monthly data, July 1951 – December 2017. HAC standard errors in parentheses. One, two and three stars denote significance at the 10, 5 and 1% level.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>1–5</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS StDev&lt;sub&gt;t&lt;/sub&gt;</td>
<td>0.008***</td>
<td>0.005***</td>
<td>0.004***</td>
<td>0.003***</td>
<td>−0.003***</td>
<td>0.011***</td>
</tr>
<tr>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>R&lt;sup&gt;M&lt;/sup&gt;&lt;sub&gt;t&lt;/sub&gt; − r&lt;sup&gt;f&lt;/sup&gt;&lt;sub&gt;t&lt;/sub&gt;</td>
<td>1.183***</td>
<td>1.146***</td>
<td>1.142***</td>
<td>1.079***</td>
<td>0.915***</td>
<td>0.268***</td>
</tr>
<tr>
<td>(0.043)</td>
<td>(0.032)</td>
<td>(0.019)</td>
<td>(0.018)</td>
<td>(0.017)</td>
<td>(0.059)</td>
<td></td>
</tr>
<tr>
<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.840</td>
<td>0.879</td>
<td>0.904</td>
<td>0.922</td>
<td>0.939</td>
<td>0.266</td>
</tr>
</tbody>
</table>

Table 10: Forecasting returns by innovation activity. The table presents results of forecasting regressions of one-year value-weighted excess returns by intensity of innovation activity. Portfolios are formed based on the number of years since a firm has been publicly listed. Innovation activity is defined as the difference between the R&D expenses to GDP ratio and its one-year lagged 10-year moving average value. Annual data, 1964 – 2017. HAC standard errors in parentheses. One, two and three stars denote significance at the 10, 5 and 1% level.

<table>
<thead>
<tr>
<th>Portfolio:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>1–5</th>
</tr>
</thead>
<tbody>
<tr>
<td>IA&lt;sub&gt;t−1&lt;/sub&gt;</td>
<td>−0.071**</td>
<td>−0.066***</td>
<td>−0.044***</td>
<td>−0.021</td>
<td>−0.023</td>
<td>−0.048***</td>
</tr>
<tr>
<td>(0.028)</td>
<td>(0.025)</td>
<td>(0.016)</td>
<td>(0.023)</td>
<td>(0.019)</td>
<td>(0.017)</td>
<td></td>
</tr>
<tr>
<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.071</td>
<td>0.083</td>
<td>0.040</td>
<td>0.010</td>
<td>0.022</td>
<td>0.084</td>
</tr>
</tbody>
</table>

Table 9 reports the results. Consistent with the predictions of the model, portfolios exhibit heterogenous comovement with shocks to idiosyncratic volatility. Portfolio 1, that contains the youngest firms, is significantly positively loaded on shocks to idiosyncratic risk. The slope coefficient monotonically declines from portfolio 1 to portfolio 5. For the “1–5” long-short portfolio the loading on the shock to idiosyncratic risk is both statistically and economically significant: one standard deviation shock to idiosyncratic volatility is associated with 1.1% monthly return spread between portfolios one and five.

Table 10 reports results of forecasting regressions of portfolios’ excess returns by innovation activity. In every year t monthly portfolio returns are aggregated to the annual level. I define the forecasting variable for returns realized in year t as the difference in R&D-to-GDP ratio in year t − 1 and the one-year lag of its 10-year moving average value.
The forecasting variable is standardized to have unit variance. According to Table 10, innovation activity significantly negatively forecasts returns of portfolios of young firms. One standard deviation increase in innovation activity (relative to the 10-year moving average value) is associated with about seven percentage point decline in subsequent one-year return on portfolio 1. The predictability pattern almost monotonically declines from portfolio 1 to portfolio 5. The “1–5” spread is significantly negatively related to the lagged innovation activity. This result is consistent with the model’s prediction that the risk-premium on assets, positively correlated with innovation shocks, is lower in times of higher innovation activity.

8 Conclusion

In this paper I proposed a model of endogenous innovation with an important role for risk and asset prices. I showed that risky nature of returns on innovation can be responsible for fluctuations in spendings on innovation. When contrasted with the data on venture capital the model showed a good ability to replicate the stylized facts on VC performance. These results demonstrate that disruption risk considerations can have far reaching consequences for the VC sector and innovation more broadly. Exploring the implications of other salient features of the investments in innovation, such as lack of liquidity, longer investment horizons or learning about new technologies, in an equilibrium framework, can be a fruitful direction of further research in an attempt to better understand the asset pricing properties of innovation.
Appendix

A Empirical Appendix

Table A1: **Innovation activity and portfolio-level idiosyncratic risk**. The table presents results of linear regressions of measures $Y_t$ of idiosyncratic risk on innovation activity $IA_t$: $Y_t = \text{Const} + \beta IA_t + \gamma_1 \text{Output Gap}_t + \gamma_2 t + \epsilon_t$. Innovation activity is measured as the detrended ratio of economy-wide R&D-to-GDP ratio normalized to have zero mean and unit variance. Output gap is normalized to unit variance. CS StDev is cross-sectional standard deviation of annual returns on wealth, Quantile Range is the difference between 95%-tile and 5%-tile of annual return on wealth, CS StDev$^U$ / CS StDev$^D$ are upside/downside standard deviations of return on wealth. Dependent variables are calculated using the data on 400 members of the Forbes 400 list. HAC standard errors in parentheses. One, two and three stars denote significance at the 10, 5 and 1% level. Annual observations, 1983–2013.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>CS StDev</th>
<th>Quantile Range</th>
<th>CS StDev$^U$</th>
<th>CS StDev$^D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$IA_t$</td>
<td>0.042***</td>
<td>0.141***</td>
<td>0.018</td>
<td>0.074**</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.042)</td>
<td>(0.012)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>Output Gap</td>
<td>−0.012</td>
<td>−0.042</td>
<td>0.009</td>
<td>−0.036</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.027)</td>
<td>(0.012)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>Mean($Y_t$)</td>
<td>0.247</td>
<td>0.743</td>
<td>0.273</td>
<td>0.228</td>
</tr>
<tr>
<td>R$^2$</td>
<td>0.468</td>
<td>0.484</td>
<td>0.203</td>
<td>0.362</td>
</tr>
</tbody>
</table>


Data on corporate earnings and corporate R&D spending comes from Compustat. I restrict my sample to US firms with non-missing observations of R&D spending and EBIT (earnings before interest and tax). When calculating aggregate R&D/Earnings and Flows/Earnings ratio I use EBIT measure for Earnings. This measure is more stable compared to earnings after tax and interest, which can be close to zero or even negative, which makes empirical moments of ratios R&D/Earnings and Flows/Earnings meaningless.

In section “Evidence from the Cross-Section of Stocks” I consider portfolios of firms that are both in Compustat and CRSP. Years since IPO are determined using the IPO date reported in Compustat, or the earliest observation of a given firm in CRSP.
B Hamilton-Jacobi-Bellman Equations

Households I will suppress the subscript for time. Recall that a household’s value function is given by \( U = (\xi H w^H)^{1-\gamma}/(1-\gamma) \). Ito’s lemma applied to \( dU \) and equation (18) imply

\[
0 = \max_{c^H \geq 0, H \sigma^H, \sigma^y H} \left\{ f(c^H, U) + U'_{c^H} \xi H c^H + U'_{w^H} (r + \sigma^H + \sigma^y H \pi^y + \sigma^H \pi^J - 1) h - c^H \right\}
\]

\[
+ \frac{1}{2} U''_{c^H} (\xi H)^2 \left( (\sigma^H)^2 + (\sigma^y H)^2 \right) dt + \frac{1}{2} U''_{w^H} (w^H)^2 \left( (\sigma^H)^2 + (\sigma^y H)^2 \right) dt
\]

\[
+ U''_{\xi w} H \xi H \left( \sigma^H \sigma^w_c + \sigma^y H \sigma^y w \right)
\]

\[
+ \frac{(\xi H w^H)^{1-\gamma}}{1-\gamma} \times \left( \left(1 + \sigma^j H \right) \left(1 + \sigma^j H \right) \right) (1 - 1) dJ, \quad (A.1)
\]

where \( \mu^H = r + \sigma^H \pi + \sigma^y H \pi y + \sigma^H (\pi^J - 1) h \). The HJB is then given by

\[
0 = \max_{c^H \geq 0, H \sigma^H, \sigma^y H} \left\{ \frac{\rho}{1-\psi} \left( \frac{c^H}{\xi H} \right)^{1-\psi} - 1 \right\} + \mu^H + (r + \sigma^H \pi + \sigma^y H \pi y + \sigma^H \pi^J - 1) h - c^H
\]

\[
- \frac{\gamma}{2} \left( (\sigma^H)^2 + (\sigma^y H)^2 \right) dt + \frac{\gamma}{2} \left( (\sigma^H)^2 + (\sigma^y H)^2 \right) dt - \frac{\gamma}{\gamma} \left( \sigma^H \sigma^w_c + \sigma^y H \sigma^y w \right)
\]

\[
+ \frac{h}{1-\gamma} \times \left( \left(1 + \sigma^j H \right) \left(1 + \sigma^j H \right) \right) (1 - 1) dJ, \quad (A.2)
\]

Substituting functional forms for \( f(c, U) \) and partial derivatives of \( U \) in the above equation yields

\[
0 = \max_{c^H \geq 0, H \sigma^H, \sigma^y H} \left\{ \frac{\rho}{1-\psi} \left( \frac{c^H}{\xi H} \right)^{1-\psi} - 1 \right\} + \mu^H + (r + \sigma^H \pi + \sigma^y H \pi y + \sigma^H \pi^J - 1) h - c^H
\]

\[
- \frac{\gamma}{2} \left( (\sigma^H)^2 + (\sigma^y H)^2 \right) dt + \frac{\gamma}{2} \left( (\sigma^H)^2 + (\sigma^y H)^2 \right) dt - \frac{\gamma}{\gamma} \left( \sigma^H \sigma^w_c + \sigma^y H \sigma^y w \right)
\]

\[
+ \frac{h}{1-\gamma} \times \left( \left(1 + \sigma^j H \right) \left(1 + \sigma^j H \right) \right) (1 - 1) dJ, \quad (A.3)
\]
Managers As before I will suppress the subscript for time. Following the same steps as for households I obtain

\[
0 = \max_{\tilde{\epsilon}^M \geq 0, \tilde{\sigma}^M, \tilde{\sigma}^y, M, i_t, e_t} \left\{ \frac{\rho}{1 - \psi} \left[ \left( \frac{\tilde{\epsilon}^M}{\xi} \right)^{1 - \psi} - 1 \right] + \mu^M \right. \\
+ \left( r + (1 - \phi) \left( \tilde{\sigma}^M \pi + \tilde{\sigma}^y \pi^y + \tilde{\sigma}^J, M (\pi^J - 1) \right) + \phi (\mu_R - r - \tilde{\epsilon}^M) \right) \\
- \frac{\gamma}{2} \left( \sigma^M + (\sigma^M)^2 + (\sigma^y)^2 \right) - 2 \frac{1 - \gamma}{\gamma} \left( \sigma^M + \sigma^y \right) \\
+ \frac{\hat{\beta}(1 - \omega^I - \omega^E)}{1 - \gamma} \left( \left[ (1 + \sigma^J, M) (1 + (1 - \phi) \tilde{\sigma}^J, M) + \phi \sigma_p \right]^{1 - \gamma} - 1 \right) \\
+ \frac{\hat{\beta} \omega^E}{1 - \gamma} \left( \left[ \frac{\xi^H}{\xi^M} (1 + \sigma^J, H) (1 - \phi) (1 + \tilde{\sigma}^J, M) \right]^{1 - \gamma} - 1 \right) \\
+ \frac{\hat{\beta} \omega^I}{1 - \gamma} \left( \left[ (1 + \sigma^J, M) (1 + (1 - \phi) \tilde{\sigma}^J, M) + \phi (\lambda (1 + \sigma^J, M) - 1) \right]^{1 - \gamma} - 1 \right) \right\}. \quad (A.4)
\]

In the equation above \( \mu_R \) stands for the expected change of the continuous component of stock return (see equation 23)

\[
\mu_R = \frac{a - \ell (g, i, t) - \frac{e_t}{p_t}}{p_t} + g_t + \mu_{p, t} + \sigma \sigma_{p, t}; \quad (A.5)
\]
symbols \( \sigma^M \) and \( \sigma^y^M \) denote manager’s total exposure to shocks \( dB \) and \( dB^y \)

\[
\sigma^M = (1 - \phi) \tilde{\sigma}^M + \phi (\sigma + \sigma_p), \quad (A.6)
\]
\[
\sigma^y^M = (1 - \phi) \tilde{\sigma}^y + \phi \sigma^y_p. \quad (A.7)
\]

C Proofs

Proof of Lemma 1 Optimal consumption and investment policies are determined by HJB equations (A.3, A.4). Taking the first order condition with respect to consumption \( \tilde{\epsilon}^H \) in equation (A.3) and \( \tilde{\epsilon}^M \) in equation (A.4) one obtains equation (37). Analogously, first order conditions of households’ HJB with respect to \( \sigma^H \) and \( \sigma^y^H \) imply equation (38) for \( i = H \). The first order condition of managers’ HJB with respect to \( \tilde{\sigma}^M \) implies

\[
(1 - \phi) \pi - \gamma (1 - \phi) \left( (1 - \phi) \tilde{\sigma}^M + \phi (\sigma + \sigma_p) \right) - (1 - \gamma) \sigma^M \xi (1 - \phi). \quad (A.1)
\]

By rearranging this equation and using the definition of \( \sigma^M \) (26) we obtain

\[
\sigma^M = (1 - \phi) \tilde{\sigma}^M + \phi (\sigma + \sigma_p) = \frac{\pi + (1 - \gamma) \sigma^M}{\gamma}. \quad (A.2)
\]

Analogously, the first order condition with respect to \( \tilde{\sigma}^y \) and equation (27) imply

\[
\sigma^y^M = (1 - \phi) \tilde{\sigma}^y + \phi \sigma^y_p = \frac{\pi^y + (1 - \gamma) \sigma^y}{\gamma}. \quad (A.3)
\]
The first order condition with respect to $\iota$ in equations (A.4, A.5) implies equation (39).

\section{D Characterizing Equilibrium}

\textbf{Lemma 2.} Managers’ wealth share $x$ follows a jump diffusion

$$dx = \mu_x dt + \sigma_x dB + \sigma_y dB^y + \sigma_x^J dJ,$$

(A.1)

with coefficients defined by

$$\mu_x = x \left( \mu^M_w - \hat{c}^M - \mu_p - g - y + (\sigma + \sigma_p)^2 - \sigma^M_w (\sigma_p + \sigma) - \sigma_p \sigma + (\sigma^y_p)^2 - \sigma^y_p \sigma^y_{w,M} \right),$$

(A.2)

$$\sigma_x = x (\sigma^M_w - \sigma - \sigma_p), \quad \sigma_y = x \left( \sigma^y_{w,M} - \sigma^y_p \right),$$

(A.3)

$$\sigma_x^J = x \left( 1 - \overline{w}^I - \overline{w}^E \right) \frac{\left( 1 + (1 - \phi) \sigma_{w}^{J,M} + \phi \sigma_{p}^J \right) + \overline{w}^I \left( 1 + (1 - \phi) \sigma_{w}^{J,M} + \phi (\lambda + \sigma_{p}^I - 1) \right)}{(1 + \sigma_p^I) \left( 1 + (\lambda - 1)(\overline{w}^I + \overline{w}^E) \right)}$$

$$+ \frac{\lambda \beta \overline{w}^E}{1 + (\lambda - 1)(\overline{w}^E + \overline{w}^I)} - x. \quad \text{(A.4)}$$

\textbf{Proof of Lemma 2} Managers’ wealth share can be written as

$$x = \frac{W^M}{pQ},$$

(A.5)

where $W^M = \int_M w^M_i \, di$ is aggregated managers’ wealth and $pQ$ is the aggregate wealth in the economy, which equals the product of the price per unit of quality times the aggregate quality. All three components, $W^M$, $p$ and $Q$, in this equation follow jump-diffusions. Managers’ wealth $W^M$ follows a jump-diffusion

$$\frac{dW^M}{W^M} = -\hat{c}^M \, dt + \mu^M_w \, dB + \sigma^M_w \, dB^y + \sigma_{w}^{J,M} \, dJ.$$}

(A.6)

The equation above follows directly from the evolution of wealth of an individual manager defined in equation (24), where by $\sigma_{w}^{J,M}$ I denote the exposure to jump. Stock price per unit of quality follows a jump-diffusion specified in equation (14). Aggregate quality follows the process

$$\frac{dQ}{Q} = (g + y) \, dt + \sigma \, dB + (\lambda - 1) \left( \overline{w}^I + \overline{w}^E \right) \, dJ.$$}

(A.7)

This expression follows from equation (7) describing the evolution of individual variety taking into account the increase in quality of product lines conditional on arrival of a wave. Using the processes for $W^M$, $p$ and $Q$ and applying Ito’s formula to equation (A.5) we obtain the drift $\mu_x$ and diffusion $\sigma_x$ and $\sigma_x^y$ coefficients specified in the lemma.

I obtain the jump coefficient $\sigma_x^J$ by calculating the value $x_t$ given $x_{t-}$ conditional on jump
arrival at time $t$. Aggregate managers’ wealth conditional on jump arrival is given by

$$W^M_t = N_{t-} (1 - \omega^I_{t-} - \omega^E_{t-}) \left( 1 + (1 - \phi) \bar{\sigma}^{J,M}_{w,t-} + \phi \sigma^J_{p,t-} \right)$$

$$+ N_{t-} \bar{\omega}^I_{t-} \left( 1 + (1 - \phi) \bar{\sigma}^{J,M}_{w,t-} + \phi (1 + \sigma^J_{p,t-}) - 1 \right) + \beta \bar{\omega}^E_{t-} \lambda p_{t-} (1 + \sigma^J_{p,t-}) Q_{t-}. \quad (A.8)$$

The first line on the right-hand side accounts for the wealth of managers whose firms were not directly affected by the wave. The second line accounts for the managers of firms that succeeded in radical innovation, the third line reflects the wealth of the new managers that govern the successful entrants.\footnote{New managers are chosen at random from the set of households. Aggregate wealth of these households prior to jump arrival is negligible due to the assumption of extreme wealth differences between a firm’s market capitalization and a typical household’s wealth. Hence, wealth of these households, prior to wave arrival, is not reflected in equation (A.8).}

Aggregate quality conditional on wave arrival is given by

$$Q_t = Q_{t-} \left( 1 + (\lambda - 1) \left( \bar{\omega}^I_{t-} + \bar{\omega}^E_{t-} \right) \right), \quad (A.9)$$

Price per unit of quality is given by

$$p_t = p_{t-} \left( 1 + \sigma^J_{p,t-} \right). \quad (A.10)$$

Jump coefficient $\sigma^J_{x,t-}$ equals to the change in managers’ share $x_t - x_{t-} = W^M_t / (p_t Q_t) - W^M_{t-} / (p_{t-} Q_{t-})$. Substituting the expressions for $W^M_t$, $Q_t$, $p_t$ outlined above into this formula delivers the expression for $\sigma^J_x$ in the statement of the lemma.

**Market Clearing Conditions** Now I rewrite market clearing conditions formulated in Definition 1 taking into account symmetric policies chosen by firms and agents within each class. Market clearing for consumption goods (30) becomes

$$\hat{\dot{c}}^M p x + \hat{\dot{c}}^H p (1 - x) = a - \iota - \hat{\dot{z}}^I - \hat{\dot{z}}^E. \quad (A.11)$$

Market clearing conditions for exposures to diffusion risk are

$$\sigma^M_w x + \sigma^H_w (1 - x) = \sigma + \sigma_p, \quad (A.12)$$

$$\sigma^y_w x + \sigma^y_{p,H} (1 - x) = \sigma^y_p. \quad (A.13)$$

Market clearing condition for jump risk is

$$x (1 - \phi) \bar{\sigma}^{J,M}_{w} + (1 - x) \bar{\sigma}^{J,H}_{w} = (1 - x \phi) \left( (1 - \omega^E - \omega^I) \sigma^J_p + \omega^I (\lambda (1 + \sigma^J_p) - 1) - \omega^E \right)$$

$$+ (1 - \beta) \lambda (1 + \sigma^J_p) \omega^E. \quad (A.14)$$

**Solving for Equilibrium Quantities Given Functions $\xi^H(x, y), \xi^M(x, y)$** This section characterizes all equilibrium quantities given functions $\xi^H(x, y)$ and $\xi^M(x, y)$ and results we established so far. First, I characterize coefficients of the process for managers’ share $x$. 
From Lemmas 1 and 2 and market clearing (A.12) it follows that

\[ \sigma_x = x (\sigma_w^M - \sigma_p - \sigma) = x (\sigma_w^M - \sigma_w^M x - \sigma_w^H (1 - x)) = \\
= x (1 - x) (\sigma_w^M - \sigma_w^H) = x (1 - x) \frac{1 - \gamma}{\gamma} (\sigma_x^M - \sigma_x^H) = \\
x (1 - x) \frac{1 - \gamma}{\gamma} (\frac{\xi_x^M}{\xi_x^M} - \frac{\xi_x^H}{\xi_x^H}) \sigma_x. \quad (A.15) \]

In the last transition I used Ito’s lemma and denoted partial derivatives with primes. In addition, I used the fact that state variable \( y \) is not correlated with the aggregate productivity shock \( dB \). The last equation implies \( \sigma_x \equiv 0 \). The aggregate productivity shock \( dB \) does not have any effect on the managers’ share. It’s only impact is on the scale of the economy. From Ito’s lemma it also follows

\[ \sigma_y = \frac{p'}{p} \sigma_x \equiv 0. \quad (A.16) \]

Following the same steps, i.e. using Lemmas 1 and 2 and market clearing condition A.13 I obtain the expression for managers’ share loading on shock \( dB^y \)

\[ \sigma_x^y = x (\sigma_w^y^M - \sigma_p^y) = x (1 - x) (\sigma_w^y^M - \sigma_w^y^H) = x (1 - x) \frac{1 - \gamma}{\gamma} (\sigma_x^y^M - \sigma_x^y^H) = \\
x (1 - x) \frac{1 - \gamma}{\gamma} \left[ \left( \frac{\xi_x^y^M}{\xi_x^y^M} - \frac{\xi_x^y^H}{\xi_x^y^H} \right) \sigma_x^y + \left( \frac{\xi_y^y}{\xi_x^y^M} - \frac{\xi_y^H}{\xi_x^y^H} \right) \sigma_y^y \right] \quad (A.17) \]

From the last equation we can solve for \( \sigma_x^y \)

\[ \sigma_x^y = \frac{\left( \frac{\xi_x^y^M}{\xi_x^y^M} - \frac{\xi_x^y^H}{\xi_x^y^H} \right) \sigma_y^y}{x (1 - x) (1 - \gamma) - \left( \frac{\xi_x^y^M}{\xi_x^y^M} - \frac{\xi_x^y^H}{\xi_x^y^H} \right) \sigma_x^y} \quad (A.18) \]

A set of equilibrium quantities, namely price \( p(x, y) \), investment in radical innovation by incumbents \( z^I(x, y) \) and entrants \( z^E(x, y) \), price of jump risk \( \pi^J(x, y) \) and managers’ exposure of liquid wealth to jump \( \sigma_{J,M}^J(x, y) \) are determined by the following set of non-linear equations

1. Market clearing of goods (A.11), where \( \hat{c}^H, \hat{c}^M \) are expressed through \( \xi^H \) and \( \xi^M \) correspondingly, and \( \iota \) is expressed as a function of \( p \) implied by the equation \( g'(\iota) = p \).

2. Equilibrium inflow in the innovation sector (44).

3. First order conditions of the managers’ HJB with respect to jump exposure \( \sigma_{J,M}^J(x, y) \) and optimal investment in radical innovation \( \hat{z}^I(x, y) \).

4. Market clearing for jump risk (A.14) with \( \sigma_{J,H}^J(x, y) \) defined by equation (41).

5. Loading on jump \( \sigma_p^J(x, y) \) is determined by \( \sigma_p^J(x, y) = (p(x(1 + \sigma_x^J), y) - p(x, y)) / p \); loadings on jump \( \sigma_x^J, \sigma_x^M \) are defined in the same manner, where \( \sigma_x^J \) is specified in Lemma 2.

The system of the above listed equations is solved numerically iteratively.
Having $p(x, y)$ in hand we obtain

$$\sigma_p^y = \frac{p_x^y}{p} \sigma_x^y + \frac{p_y^y}{p} \sigma_y^y. \quad (A.19)$$

By rearranging equations (A.3) we obtain managers’ portfolio loadings on the two types of diffusion risk

$$\sigma_w^M = \sigma_p + \sigma + \frac{\sigma_x}{x} = \sigma, \quad \sigma_{w, y}^M = \sigma_p^y + \frac{\sigma_y^y}{y}. \quad (A.20)$$

Substituting these values into the optimal portfolio choice, specified in Lemma 1, I solve for the prices of diffusion risk

$$\pi = \gamma \sigma_w^M + (\gamma - 1) \sigma \xi, \quad \pi^y = \gamma \sigma_{w, y}^M + (\gamma - 1) \sigma_y \xi. \quad (A.21)$$

**Numerical Algorithm** I modify HJBs of households and managers to allow for time-dependence in $\xi^M$ and $\xi^H$. In addition, I differentiate the market clearing condition (A.11) with respect to time allowing for time dependence in $p$, in addition to $\xi^M$ and $\xi^H$. The obtained system of partial differential equations for $\xi^M$, $\xi^H$, $p$ is iterated backward in time using the explicit scheme on a grid for $(x, y)$ starting from an initial guess for functions $\xi^M(x, y)$ and $\xi^H(x, y)$. At every iteration, given functions $\xi^M$ and $\xi^H$, I solve for all other equilibrium quantities following the procedure described above. The backward iterations are conducted till updates to functions $\xi^M(x, y)$ and $\xi^H(x, y)$ do not exceed a pre-specified threshold.
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